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Abstract

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Public good provision financed by nonlinear income tax under reduction of envy *

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JEL Classification: D63, H21, H41

Keywords: Income taxation, Public good provision, Envy free, Intensity of envy, Elasticity of substitution

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1. Introduction

In an economy where agents have different skill levels, there are several ethical reasons to consider redistribution. One of them is envy caused by income inequality. An agent envies the other agent if he prefers the other's commodity bundle to his own. We call envy-free allocation where there is no envy for every agent. Income inequalities or lots of complaints among citizens lead to producing collective decision out of the way. As seen in reality, Brexit or other electoral consequences like Donald Trump elected as the president of the United States reveal anti-globalism, and some of specialists claim that one of the reasons is to remove envy of the poor to the rich ones.[1] Also, as Bös and Tillmann (1985) noted;

> *the economic rationale for a minimization or reduction of envy by taxation is the following. Excessive envy in a society is an element of social disorder. Reducing envy in a society is a step towards increasing social harmony.*

So, reducing envy is not only a normative concept like left-wing views, but also a relevant constraint for politicians concerned with the harmony of society so as to avoid any miserable results of referendum.

In the context of income taxation with endogenous labor supply, high-skilled agents cannot envy low-skilled ones because of self-selection constraint; on the other hand, low-skilled agents must envy high-skilled ones. It is difficult to apply the original envy-free constraint in Varian (1974), but we replace the weaker and cardinal criterion proposed by Diamantaras and Thomson (1990) to evaluate the intensity of envy, called λ-envy free, and examine the optimal policy schedule under not only self-selection or incentive compatibility which extracts true information about skill from each agent, but also reduction of envy constraints.

Apart from public finance models, there are several ways to redistribute collected incomes from rich ones to poor ones in reality. For instance, the government levies taxes on workers' incomes, and transfers the wealth from rich to poor. Another is to provide public services which are necessary for everyone, but not provided by private sector, or people with low income would have limited access to such services if the government did not provide them. With reducing complaints between members in society, the policymaker sets the optimal policy for income redistribution and implementation of such public project. Also, since pro-

[1] For example, according to World Economic Forum (2017), income gap is one of major sources bringing about polarized political outcomes.

viding public service is one of the redistribution schemes, requirement of reducing envy affects her decision about it. For example, in OECD (2015), Gini indexes in 2015 of Scandinavian countries (Sweden, Norway, Finland and Denmark), where people put more weights on egalitarianism as social justice, are lower than those average in the 30 OECD member countries, and the general government spending in 2015 for social protection in these countries are in high level compared to the OECD average.[2]

In this paper, we investigate the optimal nonlinear income taxation with public good provision constrained on reduction of envy as well as conventional constraints used in Boadway and Keen (1993). The objective of the government is to achieve Pareto efficient allocation, so it maximizes the low-class utility given requirements for high-class utility, budget constraint, self-selection and reduction of envy. In this case, we derive optimal provision rule of public good as well as marginal income tax rate for each class. About marginal tax rate, we obtain the same as Nishimura (2003b); on the other hand, we derive the optimal provision rule parallel to Boadway and Keen (1993) except for distortion arisen from λ envy free constraint. This ethical constraint for low class allows the policymaker to compare the marginal rate of substitution for high class and that for λ high class, and she makes use of the difference in order to relax that constraint. Especially, because changing the amount of private consumptions for high type implies changing that for λ high type λ times as much as original high type, the direction of distortion is determined by whether the marginal rate of substitution is step-up or step-down. For the purpose of digesting the provision rule more clearly, we use constant elasticity of substitution utility function on private consumption and public good, and show that the elasticity of substitution plays a key role in determining the sign. In addition, we conduct the numerical simulation in order to unveil the effect of λ on public provision. As the extensions, we study public good provision under mixed taxation, keeping the other settings.

Related Literatures

We tick off papers related to this project which is categorized into taxation with public good provision and optimal taxation under reduction of envy. In advance, some of readers think that the latter means taxation under cases where a policymaker has objective satisfying fair

[2]See OECD (2017).

distributive taste like maximin or Pigou-Dalton principle.[3] However, this paper allows her to set reduction of envy as one of the constraints, not objective. With regard to optimal taxation for reduction of envy, Nishimura (2003b) studies optimal nonlinear income taxation under constraints about reduction of envy, which shows that the marginal tax rate can increase only if leisure is a luxury. Also, Nishimura (2003a) examines optimal commodity taxation for reduction of envy. Both papers adopt particular envy-free notion, λ envy free by Diamantaras and Thomson (1990). We follow this manner, but our paper introduces public good provision by the government which differs from these two papers.

As to optimal nonlinear income taxation with public good provision, Boadway and Keen (1993) investigates optimal income taxation with pure public good provision, and they show that a government provides a public good following modified Samuelson rule which embraces self-selection term. It means that the policymaker reduces the provision level when the valuation of agents mimicking low-skill is greater than that of mimicked low-skilled ones so as to redistribute more taxed wealths. Nava et al. (1996) studies optimal nonlinear income taxation and linear commodity taxation with pure public good provision, and with regard to the provision rule, they show that Samuelson rule is modified by two additional terms related to the self-selection constraint and to the revenue of indirect taxes, and that these terms vanish when the utility function is weakly separable between public and private goods (taken together) and leisure. Gaube (2005) provides sufficient condition for both a lower and a higher level of public expenditures in the second best than in the first best based on Boadway and Keen (1993). Recently, Aronsson and Johansson-Stenman (2008) and Micheletto (2011) study public good provision under optimal nonlinear income tax in settings where every taxpayer cares for the others' consumptions in some sense, but these works incorporate other regarding components into taxpayers' utility, which is different from ours.

This remainder of this paper is organized as follows. Section 2 examines the optimal provision rule for pure public goods under the reduction of envy, and section 3 presents simple numerical examples. Section 4 extends to the model with linear commodity taxation, and section 5 offers concluding remarks.

[3] For instance, Fleurbaey and Maniquet (2006) derives the optimal income tax schedule in settings where social planner maximizes social index satisfying several axioms for fairness and inequality aversion. They characterize the social index meeting several axioms before deriving the optimal policy.

2. Optimal income taxation with public good provision for reduction of envy

We consider a two-class economy in which each agent ($i = H, L$) possesses an exogenous skill level w_i, where $w_H > w_L > 0$. There is a continuum of individuals with unit mass. Let $n_H \in (0, 1)$ denotes high-skilled individuals and the remaining $n_L = 1 - n_H$ denotes low-skilled ones. They earn their income by labor supply, and their earnings are the product of unit wage (or skill level) and the amount of labor supply. The government collects taxes on their income, and she can schedule it nonlinearly. In addition, she provides public good by collected taxes.

At first, we assume three kinds of goods, consumption (or after-tax income) $c \in \mathbb{R}_+$, labor supply l, and public good $G \in \mathbb{R}_+$. Also, it is assumed that each worker provides the amount of labor at most $\bar{l}$, so he chooses the supply level l between 0 and $\bar{l}$. Every agent shares identical utility function $U(c_i, G, l_i)$, and U is twice continuously differentiable, strictly concave, and strictly increasing in c and G and strictly decreasing in l. Let Y be the labor income, and if agent i with skill w_i earns labor income Y_i, we can replace the expression with $U(c_i, G, \frac{Y_i}{w_i})$. In providing public good, the government must incur its production cost $\phi(G)$ with an strictly increasing, strictly convex, and twice continuously differentiable function. For all goods except for public good, a good with subscript i means the one which agent i enjoys.

Here, we assume that the government wants to achieve constrained Pareto-efficient allocation, so under several constraints with high-skilled agents having at least a given utility level $\bar{u}$, she wants to maximize low-skilled utility. It is obvious that she faces the resource constraint. Let $T : \mathbb{R} \to \mathbb{R}$ be the income tax function, and agent i's budget constraint is written as $c_i = w_i l_i - T(w_i l_i)$. So, the government's resource constraint is

$$n_L T(w_L l_L) + n_H T(w_H l_H) = n_L(w_L l_L - c_L) + n_H(w_H l_H - c_H) \geq \phi(G). \tag{1}$$

It is natural that the policymaker cannot observe agents' skill directly but their earned income, so we require that she resolves the information asymmetricity called *self-selection* constraint. We formulate it as follows:

$$U(c_i, G, l_i) \geq U(c_j, G, \frac{w_j}{w_i} l_j) \tag{2}$$

for any $i, j = H, L$ with $i \neq j$. Finally, we impose ethical constraint for reducing envy. The celebrated equity concept of no-envy faces the difficulty in the second best situation since low-skilled agent always envies high skilled agent while high-skilled agent never envies the low-skilled agent.[4] As an alternative approach, we adopt λ envy free introduced by Diamantaras and Thomson (1990) and used in Nishimura (2003a,b) as a cardinal measure for the intensity of envy.[5] Let λ_{ij} be a non-negative real number such that $U(c_i, G, l_i) = U(\lambda_{ij} c_j, G, \bar{l} - \lambda_{ij}(\bar{l} - l_j))$ when $U(c_i, G, l_i) \leq U(c_j, G, l_j)$, and $\lambda_{ij} \equiv 1$ when $U(c_i, G, l_i) > U(c_j, G, l_j)$. If λ_{ij} is unity, it is the no envy case. When agent i envies agent j, the value of λ_{ij} represents the amount by which one would have to shrink j's bundle to stop envying agent j for agent i, that is, λ_{ij} indicates the intensity of envy. Suppose that an agent i compares i's own bundle with a proportional contraction of agent j' bundle between the point $(0, \bar{l})$ and (x_j, l_j). Let $\lambda \equiv \min_{ij} \lambda_{ij}$. Under binding self-selection constraint, $\lambda = \lambda_{LH}$ since $\lambda_{LH} < 1$ and $\lambda_{HL} = 1$. An allocation is λ envy free if $U(c_i, G, l_i) \geq U(\lambda c_j, G, \bar{l} - \lambda(\bar{l} - l_j))$ for all i and j. We consider that the government is constrained by a given λ envy free requirement:

$$U(c_i, G, l_i) \geq U(\lambda c_j, G, \bar{l} - \lambda(\bar{l} - l_j)) \tag{3}$$

for any $i, j = H, L$ with $i \neq j$.[6] Due to the fact that high-skilled agent never envies low-skilled agent, we focus only on the λ envy free constraint for low-skilled agent.

Summarizing the above, we write down the policymaker's optimization problem as follows:

$$\max_{\{c_i, l_i\}_{i=L,H}, G} U(c_L, G, l_L)$$

[4] By self-selection constraint for high-skilled agent, the following inequality holds:

$$U(c_H, G, l_H) \geq U(c_L, G, \frac{w_L}{w_H} l_L) > U(c_L, G, l_L)$$

Therefore, high-skilled agent never envies the low-skilled agent, which means that envy free constraint for low-skilled agent is not satisfied.

[5] We use cardinal concepts, not ordinal concepts. According to Bös and Tillmann (1985), ordinal concepts are not useful because there is an invariant hierarchy of envy under the second best analysis. Also, the reason we adopt λ envy free as cardinal concepts is that the approach is independent of comparability and cardinality of utility functions. For example, Varian (1976) considers the value of $u(x_j) - u(x_i)$ as a cardinal measure of envy.

[6] Nishimura (2000) presents the tax policy implications under the Pareto efficient allocations which maximize λ as in Diamantaras and Thomson (1990). He shows that envy is minimized at the leximin allocation which maximizes the utility of low-skilled agent. In contrast, this paper examines the second best Pareto efficient allocations corresponding to various λ as in Nishimura (2003a,b).

subject to

$$U(c_H, G, l_H) \geq \bar{u}$$

$$n_L(w_L l_L - c_L) + n_H(w_H l_H - c_H) \geq \phi(G)$$

$$U(c_i, G, l_i) \geq U(c_j, G, \frac{w_j}{w_i} l_j) \quad \text{where } i, j = H, L \text{ with } i \neq j$$

$$U(c_L, G, l_L) \geq U(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))$$

The Lagrangian is

$$\begin{aligned} &\mathcal{L}(c_L, c_H, l_L, l_H, G; \gamma, \delta_r, \delta_{sH}, \delta_{sL}, \delta_e) = \\ &U(c_L, G, l_L) + \gamma\{U(c_H, G, l_H) - \bar{u}\} \\ &+\delta_r\{n_L(w_L l_L - c_L) + n_H(w_H l_H - c_H) - \phi(G)\} \\ &+\delta_{sH}\{U(c_H, G, l_H) - U(c_L, G, \frac{w_L}{w_H} l_L)\} + \delta_{sL}\{U(c_L, G, l_L) - U(c_H, G, \frac{w_H}{w_L} l_H)\} \\ &+\delta_e\{U(c_L, G, l_L) - U(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))\} \end{aligned} \tag{4}$$

where $\gamma, \delta_r, \delta_{sH}, \delta_{sL}$ and δ_e are Lagrangian multipliers associated with the first, second, third, fourth and fifth constraints individually.[7] Note that this problem is almost the same as Boadway and Keen (1993), but differs in the constraint of λ envy free. The first-order conditions with respect to the Lagrangian are shown in Appendix A. Hereafter, we focus on redistributive cases only; $\delta_{sL} = 0$ and $\delta_{sH} > 0$. Also, we want to consider cases where all these constraints are binding, so λ is close to 1 (but not equal to 1), and we set λ which satisfies

$$U(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H)) > U(c_H, G, \frac{w_H}{w_L} l_H).$$

If it is not satisfied, the λ envy free constraint for low-skilled agent is not binding from $\delta_{sL} = 0$.

2.1 Marginal Income Tax Rate

We check the marginal income tax rate. Basically, we derive them in the same way as Nishimura (2003b). Let $U_a^i \equiv \partial U(c_i, G, l_i)/\partial a$, $\hat{U}_a \equiv \partial U(c_L, G, \frac{w_L}{w_H} l_L)/\partial a$, and $\bar{U}_a \equiv \partial U(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))/\partial a$, where $i = H, L$ and $a = c, l$. Next lemma gives the marginal tax rates.

[7] Nishimura (2003a) demonstrates that the second-best frontier with λ envy free constraint gradually shrinks as λ increases. Indeed, as long as δ_e is positive, $\mathcal{L}$ decreases λ.

Lemma 1. *Under redistributive cases that $\delta_{sL} = 0$ and $\delta_{sH} > 0$,*

1. *Marginal income tax rate at the bottom*

$$T'(w_L l_L) = \frac{\delta_{sH}\hat{U}_c}{\delta_r}\left[MRS^L(y,c) - M\hat{R}S(y,c)\right] > 0$$

where $MRS^L(y,c) = -\frac{1}{w_L}\frac{U_l^L}{U_c^L}$ *and* $M\hat{R}S(y,c) = -\frac{1}{w_H}\frac{\hat{U}_l}{\hat{U}_c}$

2. *Marginal income tax rate at the top*

$$T'(w_H l_H) = \frac{\lambda\delta_e\bar{U}_c}{\delta_r w_H}\left[MRS_{lc}^H - M\bar{R}S_{lc}\right]$$

where $MRS_{lc}^H \equiv -\frac{U_l^H}{U_c^H}$ *is the marginal rate of substitution for* l_H *measured by* c_H*, and* $M\bar{R}S_{lc} \equiv -\frac{\bar{U}_l}{\bar{U}_c}$ *the marginal rate of substitution measured at* $(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))$.

This is consistent with Nishimura (2003b).[8] For the marginal income tax rate for the low-skilled agents, it must be positive due to self-selection constraint for agents with high skill, as shown by Stiglitz (1982). On the other hand, the marginal tax rate on the top is different from the standard result presented by Stiglitz (1982) since the term which represents the effect to λ envy free constraint appears.[9] Nishimura (2003b) shows that if the income elasticity of leisure is greater (less) than 1, MRS_{lc}^H is greater (less) than $M\bar{R}S_{lc}$, which means that the marginal income tax rate on the top must be positive (negative).[10] Of course, if $MRS_{lc}^H = M\bar{R}S_{lc}$, it must be zero. Also, if the equitability constraint does not bind, in other words, $\delta_e = 0$, then it must be 0.

2.2 Provision rule of public good

This section exhibits public good provision rule at optimum. As seen in Boadway and Keen (1993), the optimal provision rule includes self-selection term, which amounts to playing an important role in redistribution. If the mimicker puts more weight on public good based on

[8]Nishimura (2003b) also examines these marginal tax rates under when self-selection constraint for low-skilled workers binds.

[9]Note that the difference in the marginal rate of substitution between consumption and labor, not efficiency-unit labor, between the envying and the envies agent is useful information to the government since λ envy free constraint has us consider a proportional shrinkage of the envied agent's bundle.

[10]According to the definition of Nishimura (2003b), if the income elasticity of leisure is greater (less) than 1, leisure is called luxury (necessity).

private consumption than mimicked one with low-skill, then the government should reduce its production and transfer the tax revenue to agents in low-class. In addition, to relax the λ envy free constraint, the government increases or decreases the amount. For instance, if the evaluation of public good for private good at the λ-scaled bundle $(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))$ is higher than that at the one high-skilled agent receives, then she must reduce the provision level in order to redistribute more incomes.

Let $U_G^i \equiv \partial U(c_i, G, l_i)/\partial G$, $\hat{U}_G \equiv \partial U(c_L, G, \frac{w_L}{w_H} l_L)/\partial G$, and $\bar{U}_G \equiv \partial U(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))/\partial G$, where $i = H, L$. Formally, we can derive the optimal rule with respect to public good provision in the next proposition.

Proposition 1. *Under nonlinear optimal income tax with λ envy free as well as self-selection constraints, the optimal provision rule is characterized by:*

$$\sum_{i=L,H} n_i MRS_{Gc}^i + \frac{\delta_{sH}}{\delta_r} \hat{U}_c (MRS_{Gc}^L - \hat{MRS}_{Gc}) + \frac{\lambda \delta_e}{\delta_r} \bar{U}_c (MRS_{Gc}^H - \frac{1}{\lambda} \bar{MRS}_{Gc}) = \phi'(G). \quad (5)$$

where $MRS_{Gc}^i \equiv \frac{U_G^i}{U_c^i}$ is the type i's marginal rate of substitution (abbreviated by MRS) for G measured by c_i, $\hat{MRS}_{Gc} \equiv \frac{\hat{U}_G}{\hat{U}_c}$ is the mimicker's MRS between c and G and $\bar{MRS}_{Gc} \equiv \frac{\bar{U}_G}{\bar{U}_c}$ is the MRS measured at $(\lambda c_H, G, \bar{l} - \lambda(\bar{l} - l_H))$.

The first term is the sum of agent i's marginal rate of substitution for public good G measured by private consumption c_i, and the second term is the effect by incentive constraint. The third term is the novel one, which reflects the effect on λ envy free constraint, and the implication is similar to that of incentive constraint. Because λ distorts the consumption-leisure bundle for the envying agent, this term may not be zero. In order to relax the λ-envy free constraint, the government changes the provision level of public good and makes room for improving welfare. We suggest an intuitive interpretation for the third term. Starting from the original Samuelson rule and consider the following redistribution: the government imposes an additional tax liability MRS_{Gc}^i on type-i individuals to increase G. The tax reform does not change the welfare of type-i individuals and the government's budget. The valuation for G of the envying agent is expressed by $\frac{1}{\lambda}\bar{MRS}_{Gc}$. If $MRS_{Gc}^H > \frac{1}{\lambda}\bar{MRS}_{Gc}$, the third term in equation (5) suggests that the original Samuelson rule should be upwardly shifted. This implies that an increase of G mitigates the intensity of envy for low-type agents because the tax liability of the envied agent is larger than that of the envying agent and then the difference of utilities between them is reduced. Therefore, the upward distortion relaxes the λ envy free constraint for low-type individuals.

Boadway and Keen (1993) shows that the original Samuelson rule for public good provision is replicated when each agent's preference is represented by $U(H(c,G),l)$, *i.e.*, c and G are weakly separable with l in the utility function. In this case, while the second bracket in the left hand side is zero, it is ambiguous whether the third bracket is zero. Next subsection derives the direction of distortion in this rule using a concrete utility function.

2.3 A special case: CES utility function

In order to examine the direction of distortions, we check a utility function which is weakly separable between labor and other variables and the utility term has constant elasticity of substitution (CES). Let the utility function be $H(c,G) = (\alpha c^\rho + \beta G^\rho)^{\frac{1}{\rho}}$ where $\rho \leq 1$. If ρ is zero, it corresponds to the Cobb-Doudlas expression $H(c,G) = c^\alpha G^\beta$. In this case, the round bracket can be represented by

$$MRS^H_{Gc} - \frac{1}{\lambda}\bar{MRS}_{Gc} = (1-\lambda^{-\rho})\left(\frac{\beta}{\alpha}\right)\left(\frac{c_H}{G}\right)^{1-\rho}.$$

$1-\lambda^{-\rho}$ determines the sign, and the elasticity of substitution $\frac{1}{1-\rho}$ plays a crucial role since $\lambda < 1$. If $\frac{1}{1-\rho} \in (0,1)$, then the direction of distortion is positive; otherwise, that direction is negative except for $\frac{1}{1-\rho} = 1$. If $\frac{1}{1-\rho} = 1$, the bracket equals 0, so the third term as well as the second term vanish. To sum up, next corollary describes the direction of distortion on provision rule.

Corollary 1. *Assume that all agents have the following utility function:* $H(c,G) = (\alpha c^\rho + \beta G^\rho)^{\frac{1}{\rho}}$. *The optimal provision rule distorts*

- *downwardly if the elasticity of substitution* $\frac{1}{1-\rho} \in (1,+\infty)$ *or* $\rho = 1$,
- *upwardly if the elasticity of substitution* $\frac{1}{1-\rho} \in (0,1)$.

In addition, the rule coincides with Samuelson rule if the elasticity of substitution equals 1.

If the elasticity of substitution is less than 1, the government reduces the private consumption for high type to increase the provision level of public good. The elasticity of substitution means the variation of the ratio between private consumption and public good when the corresponding marginal rate of substitution changes. So, when the elasticity of substitution is less than 1, taxpayers are willing to increase the level of public good whereas decrease private consumption, and the volume of public good is greater than that of the private. Thus, the government must increase the amount of public good and reduce private consumption.

The validity of Samuelson rule under $\rho = 0$ stems from that $H(\cdot)$ is homothetic in c. In this case, a proportional shrinkage of the envied agent's consumption implies that the marginal rate of substitution decreases proportionally as c decreases. Thus, $MRS^H_{Gc} = \frac{1}{\lambda}\bar{MRS}_{Gc}$ holds.[11] What's more, with respect to λ, in the case of increasing the provision level, if $\lambda(1-\lambda^{-\rho})$ is increasing in λ, the provision level should be increased, too. For example, if ρ is sufficiently small, this part must be increasing in λ, so the government provides more public good. It means that she increases the level by collecting more income taxes if the government has to care for envy more sensitively. However, it is complex to find the effect of λ on public provision when ρ is positive or negative but close to 0. That is left for future research, but as one of the illustrations, we exhibit the numerical simulation results, which is consistent with the above implication.

3. Numerical examples

In this section, we conduct a quantitative analysis of the social welfare and the amount of the public good. In addition, we present the sensitivity of the social welfare and the amount of the public good with respect to changes in the parameter value expressing the intensity of envy, i.e., λ.

In the simulation, we set the following assumptions. First, we assume that the disutility of labor $v(\cdot)$ takes an isoelastic form: $v(\ell_i) = \ell_i^{1+1/e}/(1+1/e)$, where $e > 0$. According to empirical estimates (see, e.g., Chetty et al. (2011)), we set $e = 2$. Second, we consider that the sub-utility function $H(\cdot)$ takes the CES form to assess our theoretical results, where $\alpha = \beta = 0.5$. Third, Fang (2006) and Goldin and Katz (2007) estimate that the college wage premium is approximately 60%. We normalize low-type individuals' parameter w_L to equal 1, and thus high-type individuals' one is assumed to be $w_H = 1.6$. Fourth, according to the OECD (2010) reports, approximately one-quarter of all adults have attained tertiary

[11] Consider that $H(\cdot)$ is homogeneous degree of k in c. The third term can be rewritten as follows:

$$\frac{\delta_e}{\delta_r}\lambda\bar{U}_c\left(\frac{H_G(G,c_H)}{H_{c_H}(G,c_H)} - \frac{H_G(G,\lambda c_H)}{\lambda H_{c_H}(G,\lambda c_H)}\right) = \frac{\delta_e}{\delta_r}\lambda\bar{u}^H_c\left(\frac{H_G(G,c_H)}{H_{c_H}(G,c_H)} - \frac{\lambda^k H_G(G,c_H)}{\lambda\times\lambda^{k-1}H_{c_H}(G,c_H)}\right) = 0$$

Therefore, we obtain $MRS^H_{Gc} = \frac{1}{\lambda}\bar{MRS}_{Gc}$.

education. Therefore, we assume that 25% of individuals are high skilled workers, that is, we set $n_H = 0.25$ and $n_L = 0.75$. Finally, we assume that $\bar{u}$ is unity and the cost function is the following form satisfying strictly increasing and strictly convex: $\phi(\cdot) = G^2$.

We suggest numerical examples under two cases: $\rho = 1$ and $\rho = -1$. Table 1 presents the case in which the original Samuelson rule is downwardly distorted. First of all, the utility level for low-skilled agent decreases as λ increases since the second best frontier shrinks. Second, the provision level decreases when the λ envy free constraint is binding. Third, the provision level decreases as λ increases. The intuition is that the government reinforces the distortion on the provision level since the redistribution schemes are restricted due to the tighten envy-free constraint as the intensity of envy increases. Table 2 describes the case in which the original Samuelson rule is upwardly distorted. As with results in Table 1, the low-skilled utility decreases as λ increases. In contrast, the provision level increases under the reduction of envy and the level increases much more as λ increases. That is, the government reinforces the redistribution by the public good provision rather than the income redistribution since the implementable schedules are limited.

4. Extension

In this section, we examine the optimal provision rule for public goods when the government employs not only labor income taxes but also commodity taxes. We assume that the government can only levy linear commodity taxes since it cannot observe individuals' consumption levels.

Again, we define the identical utility function of agent i as the following form: $U(c_i, x_i, G, l_i)$, where c_i is a numeraire commodity and x_i another commodity. Producer price of commodity x are constant and normalized to unity for simplicity. While the government cannot impose any taxes on the numeraire good, it imposes proportional commodity tax t on x_i. For simplicity, we assume that $n_H = n_L = 1$ which does not affect the tax schedules crucially. The other notations are followed by the above section.

Following Mirrlees (1976) and Jacobs and Boadway (2014), we decompose individual optimization into two stages. At the first stage, each agent chooses the amount of labor supply given nonlinear income taxes, which leads to determine disposable income $R_i \equiv w_i l_i - T(w_i l_i)$. At the second stage, each agent expenses disposable income to consume a numeraire and another commodity. We suppose that individuals anticipate the outcome of the second stage at the first stage. Now, we formally turn to the analysis of individuals' problem. In the second stage, given $\{p, R_i, G, l_i\}$, agent i chooses c_i and x_i to maximize the utility $U(c_i, x_i, G, l_i)$ subject to the budget constraint $c_i + px_i = R_i$, where $p \equiv 1 + t$ is the consumer price with respect to another commodity. The first-order conditions with respect to c_i and x_i yield

$$\frac{U_x^i}{U_c^i} = p \tag{6}$$

The optimal solutions with respect to a numeraire and another commodity are denoted by $c_i^* \equiv c(p, R_i, G, l_i)$ and $x_i^* \equiv x(p, R_i, G, l_i)$ respectively. As a result, substituting these solution into the utility function yields the indirect utility function $V_i \equiv V(p, R_i, G, l_i) \equiv U(c_i^*, x_i^*, G, l_i)$. Let V_p^i, V_R^i, V_G^i, and V_l^i be the partial derivative of V_i with respect to p, R_i, G, and l respectively. From the Roy's identity and the Slutsky decomposition, we can get the following relationship.

$$-\frac{V_p^i}{V_R^i} = x_i^* \tag{7}$$

$$\frac{\partial x_i^*}{\partial p} = \frac{\partial \tilde{x}_i}{\partial p} - \frac{\partial x_i^*}{\partial R_i} \cdot x_i^* \tag{8}$$

$$\frac{\partial x_i^*}{\partial G} = \frac{\partial \tilde{x}_i}{\partial G} + \frac{\partial x_i^*}{\partial R_i} \frac{V_G^i}{V_R^i} \tag{9}$$

$$\frac{\partial c_i^*}{\partial p} = \frac{\partial \tilde{c}_i}{\partial p} - \frac{\partial c_i^*}{\partial R_i} \cdot x_i^* \tag{10}$$

$$\frac{\partial c_i^*}{\partial G} = \frac{\partial \tilde{c}_i}{\partial G} + \frac{\partial c_i^*}{\partial R_i} \frac{V_G^i}{V_R^i} \tag{11}$$

where $\tilde{c}_i$ and $\tilde{x}_i$ indicates the compensated demand function of individual i for numeraire and the taxable good, respectively.

In the first stage, each agent chooses the amount of labor supply to maximize the indirect utility V_i subject to $R_i = w_i l_i - T(w_i l_i)$. The first-order condition is given by:

$$-\frac{V_l^i}{w_i V_R^i} = -\frac{U_l^i}{w_i U_c^i} = 1 - T'(w_i l_i) \tag{12}$$

Proceedings as above, the government faces the budget constraint, self-selection constraint to prevent high-skilled from mimicking low-skilled, and λ-equitability constraint for reducing envy. We formulate them respectively as follows:

$$\sum_{i=H,L} [w_i l_i - R_i + (p-1)x_i^*] \geq \phi(G) \tag{13}$$

$$V(p, R_H, G, l_H) \geq V(p, R_L, G, \frac{w_L}{w_H} l_L) \equiv \hat{V} \tag{14}$$

$$V(p, R_L, G, l_L) \geq U(\lambda c_H^*, \lambda x_H^*, G, \bar{l} - \lambda(\bar{l} - l_H)) \equiv \bar{V} \tag{15}$$

To sum up, the restricted Pareto optimization problem to the government is given by:

$$\max_{\{R_i, l_i\}_{i=L,H}, p, G} V(p, R_L, G, l_L)$$

subject to

$$\begin{aligned}
&V(p, R_H, G, l_H) \geq \bar{u} \\
&\sum_{i=H,L} [w_i l_i - R_i + (p-1)x_i^*] \geq \phi(G) \\
&V(p, R_H, G, l_H) \geq V(p, R_L, G, \frac{w_L}{w_H} l_L) \\
&V(p, R_L, G, l_L) \geq U(\lambda c_H^*, \lambda x_H^*, G, \bar{l} - \lambda(\bar{l} - l_H))
\end{aligned}$$

The Lagrangian is

$$\begin{aligned}
\mathcal{L}(p, R_L, R_H, l_L, l_H, G; \mu, \gamma, \delta, \eta) =& V(p, R_L, G, l_L) + \mu\{V(p, R_H, G, l_H) - \bar{u}\} \\
&+\gamma\{\sum_{i=H,L} [w_i l_i - R_i + (p-1)x_i^*] - \phi(G)\} \\
&+\delta\{V(p, R_H, G, l_H) - V(p, R_L, G, \frac{w_L}{w_H} l_L)\} \\
&+\eta\{V(p, R_L, G, l_L) - U(\lambda c_H^*, \lambda x_H^*, G, \bar{l} - \lambda(\bar{l} - l_H))\}
\end{aligned} \tag{16}$$

where μ, γ, δ, and η are Lagrangian multipliers corresponding to constraints respectively. The first-order conditions with respect to the Lagrangian are shown in Appendix B.

Before analyzing the provision rule for public goods, it is useful to explore optimal linear commodity tax rate. Let $\hat{V}_p$, $\hat{V}_R$, and $\hat{V}_G$ be the partial derivative of $\hat{V}$ with respect to p, R, and G. The linear commodity tax rate is characterized by the following proposition.

Proposition 2. *Suppose that the allocations are restricted by the reduction of envy. The optimal commodity tax rate under nonlinear labor income tax and public goods provision is given by:*

$$t \sum_{i=L,H} \frac{\partial \tilde{x}_i}{\partial p} = \frac{\delta}{\gamma} \hat{V}_R (x_L^* - \hat{x}) + \frac{\lambda \eta}{\gamma} \left[\bar{U}_c \frac{\partial \tilde{c}_H}{\partial p} + \bar{U}_x \frac{\partial \tilde{x}_H}{\partial p} \right] \tag{17}$$

where, $\hat{x} \equiv x(p, R_L, G, \frac{w_L}{w_H} l_L)$ is the mimicker's demand for another commodity and $\bar{U}_r$ ($r = c, x$) is the derivative of r at λ-scaled bundle $(\lambda c_H^*, \lambda x_H^*, G, \bar{l} - \lambda(\bar{l} - l_H))$.

On the right hand side, the first term is self-selection effect, and if agent's utility is separable between commodity part and labor supply term, then it must vanish. We see this effect frequently in existing literatures of mixed taxation, but the second term is the original part for reducing envy as seen in Nishimura (2003a,b). Each term in the bracket is the inner product of the marginal utility of the low-skilled agent and the substitution effect of the compensated demand, which reflects the reduction of envy through discouragement of consumption by the high-skilled agent due to taxation. Moreover, the second term in the right hand side can be rewritten as follows:[12]

$$\frac{\lambda\eta}{\gamma}\left[\bar{U}_c\frac{\partial\tilde{c}_H}{\partial p}+\bar{U}_x\frac{\partial\tilde{x}_H}{\partial p}\right]=\frac{\partial\tilde{x}_H}{\partial p}\bar{U}_c\left[\frac{\bar{U}_x}{\bar{U}_c}-\frac{U_x^H}{U_c^H}\right]\equiv\frac{\partial\tilde{x}_H}{\partial p}\bar{U}_c\left[M\bar{R}S_{cx}-MRS_{cx}\right] \tag{18}$$

If the envying agent prefers the taxable good to the numeraire more than the envied agent, i.e., $M\bar{R}S_{cx}>MRS_{cx}$, it is taxed more heavily. This term remains even if the utility function is weakly separable between public and private goods (taken together) and leisure, that is, $U(H(c_i,x_i,G),l_i)$, while the first term in the right hand side in equation (17) vanishes. To replicate the Atkinson and Stiglitz (1976) theorem (hereafter A-S theorem), we assume the following functional form: $H(f(c_i,x_i),G)$, where $f(\cdot)$ is homothetic. In this case, the second term in the right hand side of equation (18) vanishes, which means that commodity taxation is superfluous. The sufficient condition to hold the A-S theorem is slightly different from Nishimura (2003a,b) since we impose the additional restriction which is the weak separability between all private consumptions and a public good.

Now, we turn to the characterization of the optimal provision rule for public goods. The optimal rule with respect to public goods provision can be derived in the next proposition.

Proposition 3. *Under linear commodity tax in addition to nonlinear income tax, the optimal provision rule taking the reduction of envy into account is characterized by:*

$$\sum_{i=L,H}\frac{V_G^i}{V_R^i}+\frac{\delta}{\gamma}\hat{V}_R\left[\frac{V_G^L}{V_R^L}-\frac{\hat{V}_G}{\hat{V}_R}\right]-\frac{\eta}{\gamma}\left[\bar{U}_c\frac{\partial\lambda\tilde{c}_H}{\partial G}+\bar{U}_x\frac{\partial\lambda\tilde{x}_H}{\partial G}+\bar{U}_G\right]=\phi'(G)-t\sum_{i=H,L}\frac{\partial\tilde{x}_i}{\partial G} \tag{19}$$

where $\bar{V}_k$ is the derivative of $\bar{V}=U(\lambda c_H^*,\lambda x_H^*,G,\bar{l}-\lambda(\bar{l}-l_H))$ with respect to $k=G,R$.

[12] Using individuals' budget constraint $c_i+px_i=R_i$, the following relationships holds:

$$\frac{\partial\tilde{c}_H}{\partial p}=-p\frac{\partial\tilde{x}_H}{\partial p}$$

In the left-hand side, the first term amounts to the sum of evaluation for public good G based on marginal utility for disposable income R, and the second term is self-selection effect. The remaining part corresponds to λ-equitability effect which is different from Nava et al. (1996). The part consists of two effects. The first is the indirect effect which is the inner product of the marginal utility of the low-skilled agent and the substitution effect of the compensated demand, which reflects the reduction of envy through discouragement of consumption by the high-skilled agent due to the provision of public good. The second is the direct effect which reduces envy by decreasing the amount of a public good. In the right-hand side, the first term is the marginal cost for public good, and the second term is analogous to Nava et al. (1996), which means the impact on indirect tax revenue in increasing the provision level. This is done through the compensated effects on consumption of the change in the level.

When can we apply the original Samuelson rule in this case? The third term in the left hand side of equation (19) can be manipulated to yield:

$$\begin{aligned}
-\frac{\eta}{\gamma}\left[\bar{U}_c\frac{\partial\lambda\tilde{c}_H}{\partial G}+\bar{U}_x\frac{\partial\lambda\tilde{x}_H}{\partial G}+\bar{U}_G\right] &= \frac{\lambda\eta}{\gamma}\bar{U}_c\left[\frac{U_G^H}{U_c^H}-\frac{1}{\lambda}\frac{\bar{U}_G}{\bar{U}_c}\right]+\frac{\lambda\eta}{\gamma}\frac{\partial\tilde{x}_H}{\partial G}\bar{U}_c\left[\frac{U_x^H}{U_c^H}-\frac{\bar{U}_x}{\bar{U}_c}\right] \\
&\equiv \frac{\lambda\eta}{\gamma}\bar{U}_c\left[MRS_{Gc}-\frac{1}{\lambda}M\bar{R}S_{Gc}\right]+\frac{\lambda\eta}{\gamma}\frac{\partial\tilde{x}_H}{\partial G}\bar{U}_c\left[MRS_{cx}-M\bar{R}S_{cx}\right]
\end{aligned} \tag{20}$$

Following by the analysis above, if the agent's utility is expressed by $U(H(c_i, x_i, G), l_i)$, then the second term in the left hand side of equation (19) which is the self-selection term vanishes. In addition, if the function H meets the following functional form: $H(f(c_i, x_i), G)$, where $f(\cdot)$ is homothetic, the second term in the right hand side of equation (20) must disappear since $MRS_{cx} = M\bar{R}S_{cx}$ holds. At the same time, the second term in the right hand side of equation (19) also vanishes since t is zero as shown above. Therefore, as in the analysis without linear commodity tax, whether to deviate from the original Samuelson rule depends on the first term in the right hand side of equation (20).

As in subsection 2.3, we investigate the direction of distortions under the case where the utility function is the CES form and has the weak separability between labor and other variables. Let the utility function be $H = (\alpha f(\cdot)^\rho + \beta G^\rho)^{\frac{1}{\rho}}$, where $\rho \leq 1$ and $f(\cdot)$ is homothetic. Under the setting, the first term in the right hand side of equation (20) can be rewritten as:

$$MRS_{Gc} - \frac{1}{\lambda}M\bar{R}S_{Gc} = (1-\lambda^{-\rho})\frac{\beta G^{\rho-1}}{\alpha f(\cdot)^{\rho-1} f_c(\cdot)}$$

Therefore, whether the original Samuelson condition is valid crucially depends on the elasticity of substitution. As with the result of Corollary 1, if the elasticity of substitution is larger (lower) than 1, the optimal provision rule distorts downwardly (upwardly), although the original Samuelson condition holds when the elasticity of substitution equals to 1.[13]

5. Conclusion

In this paper, we analyze optimal policy for income taxation with public good provision by a government when she is concerned with ethical constraint, reduction of envy. As the new constraint, she adopts λ-equitability borrowed from Diamantaras and Thomson (1990). In providing public good, we derive the optimal provision rule as well as marginal income tax rate in optimal policy. Though the income tax part is parallel to the result in Nishimura (2003a,b), the modified provision rule includes the effect of reducing envy, which is different from modified Samuelson rule in Boadway and Keen (1993). In order to relax the ethical constraint, she adjusts the amount of provided public good, comparing the evaluation for low-skilled agents with that at the referred commodity bundle. For instance, if an agent with the envied bundle puts more weight on public good than low-skilled agent, she must decrease the provision level in order to make use of more taxed incomes for redistribution. Furthermore, using CES utility for public good and private consumption, we show that if the elasticity of substitution is greater (lower) than 1, the original Samuelson condition is downwardly (upwardly) distorted. However, the original rule is valid if the elasticity of substitution is 1. As the extension, we add taxable consumption good and the linear commodity tax, and study both the optimal tax rate and provision rule of public good.

In Section 3 of numerical simulation part, there are two policy implications in our model. First of all, in paying attention to reduction of envy, the government must deal with the

[13]In general, if H is homogeneous degree of j in f on $H = H(f(c_i, x_i), G)$ and $f(\cdot)$ is homothetic under the weak separability between labor and other variables, the original Samuelson rule holds. The first term in the right hand side of equation (20) can be rewritten as:

$$\begin{aligned}&\frac{\lambda\eta}{\gamma}\bar{U}_c\left(\frac{H_G(f(c_H,x_H),G)}{H_c(f(c_H,x_H),G)f_c(c_H,x_H)} - \frac{H_G(f(\lambda c_H,\lambda x_H),G)}{\lambda H_c(f(\lambda c_H,\lambda x_H),G)f_c(\lambda c_H,\lambda x_H)}\right)\\ &= \frac{\lambda\eta}{\gamma}\bar{U}_c\left(\frac{H_G(f(c_H,x_H),G)}{H_c(f(c_H,x_H),G)f_c(c_H,x_H)} - \frac{\lambda^{kj}H_G(f(c_H,x_H),G)}{\lambda\times\lambda^{kj-1}H_c(f(c_H,x_H),G)f_c(c_H,x_H)}\right) = 0\end{aligned}$$

Therefore, λ-equitability term disappears and the original Samuelson rule is replicated.

envied λ-scale bundle relative to the original bundle. Due to that, the government decreases the public provision level when the elasticity of substitution between private consumption and public good is less than 1, *i.e.*, change in the ratio of these two willingness to pay is sensitive to the variation in the ratio of these volumes. Another is that the public provision increases much more or decreases much less as the intensity of envy increases. Since rising the degree makes tighter envy-free constraint, the policymaker cannot utilize the other redistribution scheme; instead, she reinforces the distorted direction about public provision.

In the end, there are several future works we come up with and leave. The first one is, related to numerical simulation part, that we derive the modified Samuelson rule but not provision level in general. Like Gaube (2005), there is room for deriving the provision level in general cases, divided into types of taxpayers' utility functions. Next one is to investigate the upper bound of intensity λ for binding envy-free constraint. Intuitively, there are three regions for the degree of envy λ: non-binding constraint region, binding constraint region and violating constraint region. As to exogenous index λ, it is also interesting to conduct comparative statics of public provision for λ analytically as future research.

Appendix A

Suppose that $\delta_{sH} > 0$ and $\delta_{sL} = 0$. Differentiating Lagrangian (4) with respect to c_L, c_H, l_L, l_H and G,

$$\frac{\partial \mathcal{L}}{\partial c_H} = (\gamma + \delta_{sH})U_c^H - \delta_r n_H - \delta_e \lambda \bar{U}_c = 0 \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial c_L} = (1 + \delta_{sL} + \delta_e)U_c^L - \delta_r n_L - \delta_{sH} \hat{U}_c = 0 \tag{A.2}$$

$$\frac{\partial \mathcal{L}}{\partial l_H} = (\gamma + \delta_{sH})U_l^H + \delta_r n_H w_H - \delta_e \lambda \bar{U}_l = 0 \tag{A.3}$$

$$\frac{\partial \mathcal{L}}{\partial l_L} = (1 + \delta_{sL} + \delta_e)U_l^L + \delta_r n_L w_L - \delta_{sH} \frac{w_L}{w_H} \hat{U}_l = 0 \tag{A.4}$$

$$\frac{\partial \mathcal{L}}{\partial G} = (\gamma + \delta_{sH})U_G^H + (1 + \delta_{sL} + \delta_e)U_G^L - \delta_r \phi'(G) - \delta_e \bar{U}_G - \delta_{sH} \hat{U}_G = 0 \tag{A.5}$$

Rearranging (A.1) and (A.3) yields the optimal marginal income tax rate at the top. On the other hand, we can derive the marginal tax rate on the bottom by combining equation (A.2) with equation (A.4). The provision rule for public good is obtained by substituting equation (A.1) and (A.2) into (A.5). □

Appendix B

Differentiating Lagrangian (20) with respect to p, R_L, R_H, and G,

$$\frac{\partial \mathcal{L}}{\partial p} = (1+\eta)V_p^L + (\mu+\delta)V_p^H - \delta \hat{V}_p - \eta \bar{V}_p + \gamma \sum_{i=H,L} [x_i^* + (p-1)\frac{\partial x_i^*}{\partial p}] = 0 \tag{B.1}$$

$$\frac{\partial \mathcal{L}}{\partial R_H} = (\mu+\delta)V_R^H - \eta \bar{V}_R - \gamma + \gamma(p-1)\frac{\partial x_H^*}{\partial R_H} = 0 \tag{B.2}$$

$$\frac{\partial \mathcal{L}}{\partial R_L} = (1+\eta)V_R^L - \delta \hat{V}_R - \gamma + \gamma(p-1)\frac{\partial x_L^*}{\partial R_L} = 0 \tag{B.3}$$

$$\frac{\partial \mathcal{L}}{\partial G} = (1+\eta)V_G^L + (\mu+\delta)V_G^H - \delta \hat{V}_G - \eta \bar{V}_G + \gamma \sum_{i=H,L} (p-1)\frac{\partial x_i^*}{\partial G} - \gamma \phi'(G) = 0 \tag{B.4}$$

Equation (B.1), (B.2), and (B.3) gives

$$\frac{\partial \mathcal{L}}{\partial p} + \sum_i \frac{\partial \mathcal{L}}{\partial R_i} x_i^* = 0 \tag{B.5}$$

Using equation (9), (10), (12), and $\hat{x} = -\frac{\hat{V}_p}{\hat{V}_R}$, equation (B.5) can be transformed into equation (17). In addition, substituting equation (B.2) and (B.3) into (B.4) and using equation (11) and (13), we can derive equation (19).

□

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	Low-skill utility	G
Case I		
Second best without λ envy free constraint	0.228045	0.499994
Case II		
Second best with λ envy free constraint (λ=0.89)	0.225014	0.497525
Case III		
Second best with λ envy free constraint (λ=0.91)	0.182929	0.489892
Case IV		
Second best with λ envy free constraint (λ=0.92)	0.0894965	0.450079

Table 1: Numerical examples: $\rho = 1$

	Low-skill utility	G
Case I		
Second best without λ envy free constraint	-0.406546	0.781823
Case II		
Second best with λ envy free constraint (λ=0.785)	-0.406572	0.78252
Case III		
Second best with λ envy free constraint (λ=0.79)	-0.428375	0.802776
Case IV		
Second best with λ envy free constraint (λ=0.792)	-0.481632	0.8229

Table 2: Numerical examples: $\rho = -1$