Long-term interest rates and bank loan supply: Evidence from firm-bank loan-level data

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Abstract

Based on a mean-variance model of bank portfolio selection subject to the value-at-risk constraint, we make predictions on transmission channels through which lower long-term interest rates increase bank loan supply: the portfolio balance channel, the bank balance sheet channel, and the risk-taking channel. Using a firm-bank loan-level panel dataset for Japan, we find evidence of the presence of these channels. First, an unanticipated reduction in long-term rates increased bank loan supply. Second, banks that enjoyed larger capital gains on their bond holdings increased loan supply. Further, this effect was stronger for loans to smaller, more leveraged, and less creditworthy firms.

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Evidence from firm-bank loan-level data

Abstract

Based on a mean-variance model of bank portfolio selection subject to the value-at-risk constraint, we make predictions on transmission channels through which lower long-term interest rates increase bank loan supply: the portfolio balance channel, the bank balance sheet channel, and the risk-taking channel. Using a firm-bank loan-level panel dataset for Japan, we find evidence of the presence of these channels. First, an unanticipated reduction in long-term rates increased bank loan supply. Second, banks that enjoyed larger capital gains on their bond holdings increased loan supply. Further, this effect was stronger for loans to smaller, more leveraged, and less creditworthy firms.

JEL classifications: E44, E52, G11, G21

Keywords: monetary policy, bank loan, portfolio balance channel, bank balance sheet channel, risk-taking channel, value-at-risk constraint
1. Introduction

Since the onset of the recent global financial crisis, central banks around the world have initiated unconventional monetary policies to stimulate economic activity and prevent deflation. One of the objectives of unconventional monetary policy is to reduce long-term interest rates, and a number of studies provide empirical evidence that unconventional monetary policy in advanced countries had the intended effect of lowering long-term interest rates (e.g., Fukunaga et al. 2015, Gagnon et al. 2011, Krishnamurthy and Vissing-Jorgensen 2011). It is not well understood, however, how banks’ lending behavior is affected by the decline in long-term interest rates brought about by those policy measures. In particular, while there is some evidence that unconventional monetary policy and/or lower long-term interest rates have led institutional investors to rebalance their portfolios towards riskier assets (Carpenter et al. 2015, Joyce et al. 2014), the evidence on bank loan supply is limited.

Against this background, the present study aims to provide simple but strong evidence that the decline in long-term interest rates has indeed stimulated
bank loan supply. To do so, we construct a unique and massive firm-bank loan-level panel dataset for Japan covering the period 2002–2014, which makes it possible to address the identification challenge that the effect of long-term interest rates on loan supply needs to be disentangled from the effect on loan demand by controlling for time-varying unobserved firm heterogeneity with firm-year fixed effects.

More specifically, we first construct a simple mean-variance model of bank portfolio selection subject to the value-at-risk (VaR) constraint, in which the VaR constraint is similar to that in Adrian and Shin (2011). We consider a bank that invests in two kinds of assets: loans and government bonds (“bonds” hereafter), taking the prices of those assets as given. Our simple framework predicts that a change in the price of bonds (i.e., long-term interest rates) affects bank loan supply via three transmission channels. The first channel is what we shall call the “portfolio balance channel.” Specifically, we argue that the effect of a reduction in long-term interest rates on loan supply depends on the trade-off between the “substitution effect” and the “income effect.” The substitution effect means that, in response to the decline in long-term interest rates, a bank
subject to the VaR constraint will increase its loan supply because the decrease in income from bond holdings makes it more profitable for the bank to hold loans. In contrast, the income effect means that the bank will reduce its loan supply because under the VaR constraint the decrease in income from bond holdings makes it costlier than before for the bank to hold loans. In sum, the effect of lower long-term interest rates on loan supply depends on the relative size of these two opposing effects, and a lower interest rate increases loan supply if the substitution effect is larger than the income effect. The second channel is the bank balance sheet channel. When interest rates fall and bond prices go up, a bank’s net worth increases through the capital gains on the bonds that it holds. The stronger balance sheet allows the bank to increase its loan supply. We call this the “net worth effect.” The third channel we examine is the risk-taking channel, which is closely related to the bank balance sheet channel (net worth effect). We extend our analytical framework to distinguish risky loans and safe loans, and our model predicts that in response to an increase in its net worth a bank will increase the supply of risky loans more than that of safe loans.

Based on this framework, we empirically examine whether these
effects were at work in banks’ lending behavior to Japanese firms during the period 2002–2014. More specifically, to examine the net worth effect (bank balance sheet channel), we analyze the cross-bank variation in bank net worth caused by changes in long-term interest rates, which are the same across banks, and banks’ interest rate risk exposure (i.e., bond holdings), which differs across banks. In order to identify shifts in bank loan supply we use firm-bank match-level loan data, which allow us to identify multiple loans to the same firm in the same year by different banks. Using such data and controlling for firm-year fixed effects to take firms’ unobservable loan demand into account, we examine the relationship between changes in individual firms’ loans from different banks and shocks to the net worth of these banks. In addition, to examine the risk-taking channel, we investigate whether the bank net worth effect is stronger for loans to riskier firms.

Regarding the income effect and the substitution effect (portfolio balance channel), we examine how unanticipated changes in long-term interest rates affect bank loan supply. Because changes in long-term interest rates are common across banks, we cannot empirically identify cross-bank variations in
the income and substitution effects. However, the rich panel data set used in this study allows us to examine which of these two opposing effects is dominant for all banks together while controlling for various time-varying firm and bank characteristics and time-invariant firm and bank fixed effects that might affect individual bank loan supply. In addition, we examine whether the portfolio balance channel is stronger for banks facing higher loan interest rates than those facing lower loan rates by interacting changes in long-term interest rates with bank-specific loan interest rates. Because this interaction term differs across banks, the additional analysis allows us to examine the heterogeneity among banks regarding the portfolio balance channel while controlling for firm-year fixed effects that take firms’ unobservable loan demand into account.

We obtain the following empirical results. First, we find that unanticipated reductions in long-term interest rates increased bank loan supply, which suggests that the substitution effect is indeed larger than the income effect. Our estimation shows that a 1 percentage point reduction in long-term interest rates raises the growth rate of a bank’s loan supply by 1.6 percentage points. Second, banks that enjoyed larger capital gains on their bond holdings
significantly increased their loan supply, which provides evidence that the bank balance sheet channel (net worth effect) plays a role. Based on our estimation result, a one standard deviation increase in a bank’s capital due to capital gains relative to its total assets (equivalent to a 0.18 percentage point increase in the ratio of bank capital to total assets) raises the growth rate of a bank’s loan supply by 0.8 percentage points. Given that the mean of the loan growth rate during the observation period was −5.2 percent, the substitution effect (net of the income effect) and the net worth effect are of modest but not negligible economic significance. Further empirical investigations we conduct show that the bank balance sheet channel is stronger with regard to loans to smaller, more leveraged, and less creditworthy firms, which suggests the existence of the risk-taking channel.

This study is closely related to the following two strands of literature. First, a growing number of theoretical and empirical studies examine the transmission channels of monetary policy, highlighting channels other than the standard interest rate channel. For instance, theoretical models developed by, among others, Bernanke and Gertler (1989) and Kiyotaki and Moore (1997),
show that a positive shock to a borrower’s net worth mitigates the financial frictions between the borrower and its lenders, and hence increases borrowing (firm balance sheet channel). In a similar vein, Adrian and Shin (2011), Gertler and Karadi (2011), Holmstrom and Tirole (1997), and Stein (1998) show that a positive shock to a financial intermediary’s net worth alleviates the financial frictions between the financial intermediary and its depositors, which results in the increase in its lending capacity (bank balance sheet channel) and the rebalancing of its portfolio towards riskier assets (risk-taking channel). While there are a number of empirical studies that provide evidence of the bank balance sheet channel as a transmission channel of monetary policy, most of these employ either aggregate data (e.g., Bernanke and Blinder 1992) or bank-level data (e.g., Hosono 2006, Kashyap and Stein 2000), which cannot clearly disentangle the effects of monetary policy on loan supply and loan demand. Against this background, recent studies, including Hosono and Miyakawa (2014) and Jiménez et al. (2012), have used firm-bank loan-level data to identify the effect of bank net worth induced by a change in monetary policy on loan supply. Other studies using firm-bank loan-level data to identify the effect of
monetary policy on banks’ risk-taking include Ioannidou et al. (2015), Jiménez et al. (2014), and Paligorova and Santos (2017).¹

Another recent strand of the literature investigates the effect of unconventional monetary policy on asset prices and how the induced changes in asset prices affect investors’ portfolios. As mentioned earlier, a number of empirical studies find that unconventional monetary policy reduces long-term interest rates (e.g., Fukunaga et al. 2015, Gagnon et al. 2011, Krishnamurthy and Vissing-Jorgensen 2011). In addition to the standard interest rate channel that works through changes in loan demand, lower long-term interest rates may lead investors to shift their portfolios toward assets other than long-term government bonds and boost the price of those other assets; this is the so-called “portfolio balance channel” (Joyce et al. 2014). Carpenter et al. (2015) and Joyce et al. (2014) respectively find evidence that institutional investors shifted their portfolios away from government bonds towards riskier assets in response to the Federal Reserve’s asset purchases program and the Bank of England’s

¹ To distinguish bank loan supply shocks from loan demand shocks, a growing number of empirical studies have been using firm-bank loan-level data. Examples include the studies by Khwaja and Mian (2008) and Schnabl (2012) on the supply-side impact of international financial crises, and Duchin and Sosyura (2014) and Giannetti and Simonov (2013) on the effect of public capital injections to banks during crises.
quantitative easing (QE).\textsuperscript{2} From a theoretical perspective, the portfolio balance channel may also apply to banks; however, as far as we are aware, there are few empirical studies on this issue, likely because in many countries government bonds make up only a small share of banks’ assets. However, as will be seen below, this is not the case for Japan, where the share of government bonds in banks’ portfolios has grown, while that of bank loans has stagnated.

This study is placed at the intersection of these two strands of literature.\textsuperscript{3} The key contribution of the study is that it examines different transmission channels of unconventional monetary policy simultaneously in a simple framework. As mentioned above, previous studies have examined the portfolio balance channel, the bank balance sheet channel, and the risk-taking channel of monetary policy independently. As far as we are aware, this is the first study to examine these channels concurrently by employing Japan’s unique institutional

\textsuperscript{2} Foley-Fisher et al. (2016) find evidence that the demand for riskier corporate debt by insurance companies increased in response to the Federal Reserve’s maturity extension program.

\textsuperscript{3} Note, however, that the portfolio balance channel in the present study is slightly different from that discussed in the literature on unconventional monetary policy. For example, the portfolio balance channel in Joyce et al. (2014) relies on the existence of the so-called “preferred-habitat” of different investors that may have peculiar investment motives other than expected return and risk, while the portfolio balance channel in the present study relies on the net effect of the substitution and income effects on banks’ portfolio selection under the VaR constraint.
setting, in which banks play a major role in corporate finance as well as in government bond markets.

Note, however, that instead of focusing on the effect of monetary policy on bank loan supply, the present study focuses on the effect of long-term interest rates on bank loan supply. There are two reasons for doing so. First, there is a consensus that monetary policy affects real activity through its effects on long-term interest rates, even though the particular mechanisms through which unconventional monetary policy affects long-term interest rates remains a subject of debate.\(^4\) As will be shown in our simple model in Section 3, banks determine their portfolio composition given the expected return of assets (loans and bonds in our model). For the sake of simplicity, we make no \textit{a priori} assumptions on the transmission mechanism of monetary policy to long-term interest rates. Instead, we take changes in long-term interest rates as our point of departure and examine whether we find any evidence of the portfolio balance

\(^4\) There are a number of theoretical and empirical studies that discuss whether unconventional monetary policies such as quantitative easing (QE) and the Large-Scale Asset Purchase Program (LSAP) affect long-term interest rates. For instance, Eggertsson and Woodford (2003) theoretically argue that under certain conditions a central bank’s asset purchases are irrelevant beyond their effect on private agents’ expectations about the future course of monetary policy (signaling effects). In contrast, Krishnamurthy and Vissing-Jørgensen (2013) highlight the role of the scarcity channel, in which the purchase of government bonds by central banks indeed affects bond prices (long-term interest rates).
channel. Simultaneously, we examine whether we find evidence of the bank balance sheet channel, since changes in long-term interest rates bring about capital gains or losses. Second, if we were to focus on monetary policy rather than long-term interest rates, it would be much harder – if not impossible – to disentangle the monetary policy stance and economic conditions. In addition, if a change in monetary policy and/or long-term interest rates is anticipated, there is a possibility of reverse causality, as banks and firms may well adjust their lending or borrowing prior to the change (Khawaja and Mian 2008). Thus, in order to examine the effect of long-term interest rates on bank loan supply, we need to single out exogenous and unanticipated changes in long-term rates that are orthogonal to banks’ lending behavior to avoid the endogeneity problem. To do so, we employ long-term forward interest rates as a proxy for the expected return on bonds, since changes in forward rates reflects unanticipated component of expected return on bonds, and are less likely to be affected by

5 Previous studies examining the impact of monetary policy on loan supply rely on settings where monetary policy tends to be relatively independent of economic conditions. For example, Jiménez et al. (2012), focusing on Spain, argue that the monetary policy of the ECB has been fairly exogenous for countries on the European periphery such as Spain, while Ioannidou et al. (2015) use observations for Bolivia, a country that has been characterized by a high level of dollarization and for which, as a result, monetary policy is essentially set by the Federal Reserve. Obviously, the situation in Japan is quite different, so that the strategies employed in these studies would not work in our setting.
current economic conditions than changes in spot interest rates or changes in monetary policy.

The remainder of the paper is organized as follows. Section 2 briefly describes developments in monetary policy and bank portfolios in Japan in the 2000s. Section 3 then presents our simple mean-variance model of bank portfolio selection subject to the VaR constraint, which provides empirical predictions. Next, Section 4 explains our data and sample selection, the empirical strategy we employ, and the variables, while Section 5 presents the empirical results. Finally, Section 6 concludes and discusses topics for future research.

2. Developments in monetary policy and bank portfolios in Japan

As mentioned, we use a firm-bank matched loan-level dataset that covers not only large listed firms but also unlisted small and medium-sized enterprises (SMEs) and spans the period from 2002 to 2014. The period covered by our data includes not only periods of monetary easing through unconventional policies but also a period, in the mid-2000s, when the Bank of Japan exited from quantitative easing, so that there are sufficient cyclical fluctuations in long-term
interest rates. In addition, given that Japan has a predominantly bank-based financial system, bank lending plays a prominent role in the provision of funds especially to SMEs that find it difficult to raise funds in capital markets, so that Japan provides a good case study of the impact of interest rates on bank loan supply. To provide some background for our analysis, this section briefly discusses developments in Japan’s monetary policy, interest rates, and banks’ asset portfolios in the 2000s using aggregate data.6

Following the collapse of the dot-com bubble in 2000, the BOJ embarked on its QE policy in March 2001, which set bank reserves as the policy target and introduced forward guidance using the Consumer Price Index as the instrument to tell the public under what conditions the BOJ would exit from QE.7 QE effectively lowered the short-term policy rate to zero. At the same time, the amount of JGBs held by the Bank of Japan increased substantially and long-term interest rates declined. The BOJ ended QE in March 2006 and raised the policy target rate to 0.25% in July of the same year. Following the Great

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6 Associated figures are provided in the Online Appendix A.
7 Specifically, in its policy statement on March 19, 2001, the BOJ announced that the QE will stay in place until the inflation rate measured by the CPI (excluding perishables) is expected to stabilize at more than zero percent.
Recession, the BOJ started “Comprehensive Monetary Easing” in October 2010. Under Comprehensive Monetary Easing, the BOJ purchased a variety of assets including exchange-traded funds (ETFs) and Japan real estate investment trusts (J-REITs) as well as JGBs. In April 2013, the BOJ introduced “Quantitative and Qualitative Monetary Easing (QQE),” under which it started purchasing massive amounts of JGBs including bonds with longer remaining maturities to increase the monetary base. QQE resulted in zero short-term rates and lower long-term rates.

Against this background, the ratio of Japanese banks’ bond holdings to total loans outstanding increased in the 2000s until the BOJ started QQE, which suggests that Japanese banks increased their exposure to interest rate risk. The loan growth rate was mostly sluggish except for the mid-2000s and after 2012, while the loan interest rate has been steadily declining except for a brief period in the mid-2000s. Sluggish loan growth and declining loan interest rates suggest that loan developments were largely driven by demand factors and that it is important to control for loan demand factors in identifying supply factors.8

8 It is also important to control for supply factors other than those we focus below. In particular, during the period this study focuses on, Japanese banks struggled with resolving massive non-performing loans, especially in the early 2000s, which may have affected their
3. Theoretical model

To derive theoretical predictions on the effect of long-term interest rates on bank lending, we construct a simple model of bank portfolio selection. Consider a bank that has net worth $N$. The bank originates loans $L$ and invests in bonds $B$, and obtains funds from deposits $D$. Thus, its profit function and balance sheet constraint are defined as

\[
\pi = r_L L + r_B B - r_D D \tag{1}
\]

\[
\text{s.t. } L + B = D + N \tag{2}
\]

where $\pi$ denotes the bank’s profit and $r_L$, $r_B$, and $r_D$ respectively represent the interest rate of loans, bonds, and deposits. We assume that the bank takes those interest rates as given and that $r_L$ and $r_B$ are stochastic variables. The mean and standard deviation of $r_L$ and $r_B$ are given by $(\mu_L, \sigma_L)$ and $(\mu_B, \sigma_B)$, respectively. Combining equations (1) and (2) yields

\[
\pi = (r_L - r_D) L + (r_B - r_D) B + r_D N \tag{3}
\]

We assume that the bank is risk averse and maximizes its expected profit while minimizing the volatility of its profit. More specifically, the bank’s loan supply. Regarding the effect of the bad loan problem on bank loan supply in Japan, see, for instance, Peek and Rosengren (2000) and Watanabe (2007).
The optimization problem is given by

$$\text{Max} \quad E[\pi] - \frac{\gamma}{2} \text{Var}[\pi] \quad (4)$$

where $\gamma$ is the parameter for relative risk aversion, which is assumed to be strictly positive. We also assume that the correlation between $r_L$ and $r_B$ is zero. We assume that the bank is subject to the VaR constraint. Under the VaR constraint, the bank will build its portfolio (loans and bonds) such that it would not be insolvent unless a considerable stress event materializes. More precisely, we assume that the VaR constraint is given by

$$(\mu_L - n\sigma_L - r_D)L + (\mu_B - n\sigma_B - r_D)B + r_DN \geq 0 \quad (5)$$

where the strictly positive parameter $n$ represents the largest magnitude of the stress in terms of the volatility of bank assets (loans and bonds) under which the bank is solvent, and $(\mu_L - n\sigma_L - r_D)$ and $(\mu_B - n\sigma_B - r_D)$ respectively represent the loss (negative spread) if the stress event materializes. Arranging inequality (5), we have

$$\frac{r_D - (\mu_L - n\sigma_L)}{r_D}L + \frac{r_D - (\mu_B - n\sigma_B)}{r_D}B \leq N \quad (5)'$$

Inequality (5)' shows that the bank should hold sufficient net worth (right-hand side) to absorb losses from loans and bonds under the stress event (left-hand side).
side) when constructing its optimal portfolio \((L^{**}, B^{**})\) so as to satisfy the inequality. The bank solves the maximization problem (4) subject to inequality (5)’.

The comparative statics for the effect of a decrease in bond returns \(\mu_B\) on the optimal amount of loans \(L^{**}\) are shown analytically in the Online Appendix B. Here, we only provide the intuition behind the results. Inequality (5)’ is analogous to a budget constraint in a standard consumption choice model, where the effects of a price change for one good can be decomposed into a substitution effect and an income effect. In our case, the substitution effect means that a decrease in \(\mu_B\) makes it relatively costly for the bank to invest in bonds and the bank hence increases \(L^{**}\). The income effect means that a decrease in \(\mu_B\) decreases income from government bonds, which tightens the VaR constraint and hence reduces \(L^{**}\). The bank thus reduces \(L^{**}\) in order to satisfy inequality (7)’. In sum, the effect of a decrease in \(\mu_B\) on \(L^{**}\) depends on the relative impacts of the substitution effect and the income effect.\(^9\)

The effect of an increase in \(N\) on \(L^{**}\) is straightforward: a larger \(N\)

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\(^9\) These effects do not exist without the VaR constraint. That is, when the expected return of bonds falls, the bank only reduces its amounts of bonds (and hence deposits) and does not change the amount of loans. For more details, see Appendix B.
makes inequality (7)' less binding and hence the bank increases $L^{**}$. Although our simple static model abstracts from how changes in bond interest rates affect banks’ net worth, in practice, when bond interest rates fall (and hence the price of bonds increases), banks’ net worth increases as a result of the increase in the value of their bond holdings. This increase in the value of banks’ bond holdings can be interpreted as an increase in $N$. This net worth effect corresponds to the bank balance sheet channel in the literature (e.g., Gertler and Karadi 2011, Holmstrom and Tirole 1997, Stein 1998).

Finally, in order to examine whether a bank will increase its holdings of risky assets more than of safe assets in response to a positive net worth shock (the risk taking channel), we extend our analysis to a three-asset case: safe loans, risky loans, and bonds. We assume that the return on risky loans has a higher mean and higher standard deviation than safe loans, while the Sharpe ratio of risky loans is lower than that of safe loans. The last assumption on the Sharpe ratio implies that loans are considered as risky if they do not offer sufficient excess return to compensate for their return volatility and is in line with the existing literature on the risk taking channel of monetary policy, which finds
that risky loans offer a lower risk premium (see, for instance, Ioannidou et al. 2015 and Paligorova and Santos 2017). Details of the model as well as the comparative statics for the effect of an increase in bank net worth on the amount of risky loans relative to safe loans are shown in the Online Appendix B. The comparative statics show that when a bank’s net worth increases, the bank increases risky loans more than safe loans. Thus, an increase in bank net worth induces more bank risk-taking.

4. Data, empirical strategy, and variables

4.1. Data and sample selection

To construct our firm-bank matched loan-level data, we use the database compiled by TEIKOKU DATABANK, LTD. (TDB). The TDB database, which is the main source of our dataset, contains information on listed and unlisted firms in Japan, including their characteristics (e.g., ownership structure, credit scores, etc.), their financial statements, and up to 15 financial institutions that each firm transacts with. Regarding financial institutions that a firm transacts with, the TDB database contains information on their identities and whether the bank is the main bank of a firm. The definition of the main bank is somewhat
subjective in that it is identified by each firm. In addition, and most importantly for our analysis, the TDB database allows us to identify the amount of loans outstanding provided by each bank that each firm transacts with. These firm-bank loan-level data are available for the period 2002–2014, although the number of observations for 2014 is much smaller than for the other years. Most variables in the TDB database are revised yearly, so that we use annual data for our panel.

We restrict our sample to firms for which data on (i) the total loans outstanding, (ii) the amount of loans outstanding from at least two banks, and (iii) the TDB credit score are available in the TDB database.\textsuperscript{10} For the reason explained below, we exclude from our sample firms that obtained loans from only one bank. Based on these sample selection criteria, we have 48,975 firms in total.

In addition to the TDB database, we use Nikkei Financial Quest, banks’ financial statements compiled by the Japanese Bankers Association, and banks’ annual reports to obtain bank-level data. Macroeconomic variables are obtained

\textsuperscript{10} The TDB credit score rates firms based on their business history, capital structure, size, profitability, funding status, CEO, and vitality. The score takes a value between 1 and 100, with a higher score representing a better rating.
from Nikkei Financial Quest. Regarding banks, we restrict our sample to
deposit-taking financial institutions that mainly focus on commercial banking.
To be more specific, our sample banks consist of city banks, regional banks,
second-tier regional banks, and Shinkin banks.\footnote{11} Regarding mergers and
acquisitions (M&A), we treat merged banks as distinct institutions from the
entities that were merged. Based on this procedure, we end up with observations
on 408 banks in total.

Using the firm and bank data described above, we construct an
unbalanced firm-bank matched loan-level panel that covers the period 2002–
2014. The total number of individual firm-bank loan observations for the entire
period is 379,989.

4.2. Empirical strategy

4.2.1 Main estimations

The advantage of firm-bank matched loan-level panel data is that such data
make it possible to disentangle credit supply shocks from credit demand shocks
and identify the bank balance sheet channel. For this reason, such data have

\footnote{11} We exclude long-term credit banks and trust banks, which are somewhat different from
commercial banks. For a detailed description of the type of banks in Japan, see Uchida and
Udell (2010).
been widely used in studies examining the bank balance sheet channel in the context of monetary policy (Hosono and Miyakawa 2014, Ioannidou et al. 2015, Jiménez et al. 2012; 2014, Paligorova and Santos (2017)), financial crises (Khwaja and Mian 2008, Schnabl 2012), and public capital injections to banks during a crisis (Duchin and Sosyura 2014, Giannetti and Simonov 2013). In the context of our study, the aim is to investigate the impact of changes in long-term interest rates on bank loan supply employing the model presented in Section 3. Specifically, in our analysis we focus on exogenous changes in banks’ net worth brought about by changes in the prices of bonds that banks are holding.

Suppose that changes in loans to firm $i$ by bank $j$ ($LOANS(i, j)$) are determined by macroeconomic shocks such as changes in long-term interest rates ($\Delta BOND RATE$), bank-specific loan supply shocks such as capital gains/losses due to changes in the value of bond holdings reflecting changes in interest rates ($BK_{\_CAPGAIN}(j)$), and firm-specific loan demand shocks ($F_{\_DEMAND}(i)$) such as an increase in sales growth. That is:

$$\Delta LOANS(i, j) = \alpha_0 + \alpha_1 \Delta BOND RATE + \alpha_2 BK_{\_CAPGAIN}(j)$$

$$+ \alpha_3 F_{\_DEMAND}(i) + \epsilon(i, j)$$

If $F_{\_DEMAND}(i)$ is unobservable, OLS regression yields biased estimates of
However, if we observe a change in loans to the same firm by another bank, \( j' \), we can write a similar equation:

\[
\Delta LOANS(i, j') = \alpha_0 + \alpha_1 \Delta BOND RATE + \alpha_2 BK_{\text{CAPGAIN}}(j') + \text{\( F_{DEMAND}(i) + \varepsilon(i, j') \)}
\]

Differencing the above two equations yields

\[
\Delta LOANS(i, j) - \Delta LOANS(i, j') = \alpha_2 (B - B) + \varepsilon(i, j) - \varepsilon(i, j')
\]

Thus, firm-specific demand shocks are eliminated when we difference the changes in loan amounts to the same firm provided by different banks and we obtain an unbiased estimate of \( \alpha_2 \) which captures the effect of bank-specific loan supply shocks. Note that for us to be able to estimate the above equation, a firm needs to have lending relationships with at least two banks. This is the reason that we exclude from our sample firms that obtained loans from only one bank.

Specifically, we estimate the following three types of regression equations:

\[
\Delta LOANS(i, j, t) = \beta_0 + \beta_1 \Delta BOND RATE(t - 1) + \beta_2 B + \beta_3 \text{MACRO}(t - 1) \\
+ \beta_4 \text{BANK}(j, t - 1) + \beta_5 \text{FIRM}(i, t - 1) + \eta(j) + \upsilon(i) + \varepsilon(i, j, t)
\]  

(6)
\[ \Delta \text{LOANS}(i,j,t) \]
\[ = \gamma_0 + \gamma_1 \text{BK\_CAPGAIN}(j, t - 1) + \gamma_2 \text{BANK}(j, t - 1) + \gamma_3 \text{FIRM}(i, t - 1) + \eta(j) + \nu(i) + \zeta(t) + \varepsilon(i,j,t) \]  
\[ \Delta \text{LOANS}(i,j,t) \]
\[ = \delta_0 + \delta_1 \text{BK\_CAPGAIN}(j, t - 1) + \delta_2 \text{BANK}(j, t - 1) + \eta(j) + \omega(i,t) + \varepsilon(i,j,t) \]

In equation (6), we control for the bank-level fixed effect \( \eta(j) \) and the firm-level fixed effect \( \nu(i) \) to capture bank- and firm-specific time-invariant factors.

In addition, we control for time-variant covariates, namely macroeconomic conditions (\textsc{Macro}(t - 1)), bank characteristics (\textsc{Bank}(j, t - 1)), and firm characteristics (\textsc{Firm}(i, t - 1)). We employ a one-year lag for all independent variables to avoid possible endogeneity problems. Next, in equation (7), we additionally include the year fixed effect \( \zeta(t) \). While this specification takes time-variant unobservable macroeconomic factors into account, including year fixed effects means that we cannot estimate the impact of changes in long-term interest rates, \( \Delta \text{Bondrate} \), and other macroeconomic variables. Finally, equation (8) incorporates the firm-year fixed effect \( \omega(i, t) \), which captures time-variant firm-level unobservable factors such as firm-specific loan demand that may not be fully captured by variables included in \( \text{Firm}(i, t - 1) \) in equations (6) and (7). In terms of the interpretation of our results, the coefficient
on $\Delta BOND RATE$ in equation (6) indicates whether higher long-term interest rates increase bank loan supply (through the income effect) or decrease it (through the substitution effect). Meanwhile, the impact of capital gains/losses on bank bond holdings on loan supply, $BK\_CAPGAIN$, is included in all three specifications, but our preferred specification is equation (8), where the firm-year fixed effect, $\omega(i,t)$, takes unobservable time-variant firm heterogeneity into account. The results of main estimations are presented in Section 5.1.

The empirical investigation on the bank balance sheet channel used here follows the identification strategy employed by Hosono and Miyakawa (2014) and Jiménez et al. (2012), who also used firm-bank loan-level panel data. However, our approach differs from theirs in that we use a different proxy for bank net worth shocks, namely capital gains accruing to banks through their interest rate risk exposure ($BK\_CAPGAIN(j)$), while the other two studies use the interaction term between the variable which represents the monetary policy stance (e.g., short-term interest rates) and banks’ net worth level prior to changes in monetary policy. While the interaction term used in these studies may indirectly measure the magnitude of the bank net worth effect brought about by
monetary policy shocks, we think capital gains accruing to banks through their interest rate exposures provide a much more direct measurement. Our approach is similar to that in studies examining the effect of public capital injections on bank loan supply, in that, just like those studies, we also directly examine the impact of additional net worth accruing to banks.

4.2.2 Cross-term estimations

As noted above, we cannot estimate the impact of changes in long-term interest rates, $\Delta BOND RATE$, on bank loan supply using our preferred specification, equation (8), which controls for the firm-year fixed effect. However, we can examine whether the strength of the portfolio balance channel differs across banks by interacting $\Delta BOND RATE$ with a bank characteristics variable while incorporating firm-year fixed effects, as in Hosono and Miyakawa (2014) and Jiménez et al. (2012). Therefore, we estimate the following equation:

$$
\Delta LOAN(i, j, t) = \theta_0 + \theta_1 \Delta BOND RATE(t - 1) \times BANK(j, t - 1) \\
+ \theta_2 BKCAPGAIN(j, t - 1) + \theta_3 BANK(j, t - 1) \\
+ \eta(j) + \omega(i, t) + \epsilon(i, j, t)
$$

(9)

The coefficient on the interaction term $\Delta BOND RATE \times BANK(j)$ measures the relative strength of the portfolio balance channel for banks with certain characteristics. The characteristic that we focus on is changes in bank-specific
loan interest rates, which allows us to examine whether the strength of the portfolio balance channel differs across banks. We expect that the substitution effect from bonds to loans is stronger for banks facing higher expected returns on loans so that $\theta_1$ is negative. The results are presented in Section 5.2.

In a similar vein, and more importantly, we examine the risk-taking channel by interacting $BK\_CAPGAIN$ with firm characteristics variables that represent firms’ riskiness. That is, we estimate the following equation:

$$\Delta LOANS(i,j,t) = \lambda_0 + \lambda_1 BK\_CAPGAIN(j, t - 1) \times FIRM(i, t - 1) + \lambda_2 BANK(j, t - 1) + \eta(j) + \omega(i, t) + \varepsilon(i, j, t).$$

As proxies for firms’ riskiness, we use firms’ size, leverage, and credit score. The results are presented in Section 5.3.

4.3. Variables

Definitions of the dependent and independent variables used in the estimation are presented in Table 1, while Table 2 provides their summary statistics. The dependent variable is $\Delta LOANS$, which represents the percentage change in loans to firm $i$ by bank $j$ in year $t$ from year $t - 1$ and is obtained by taking the log-difference between year $t$ and $t - 1$. We define loans as the sum of
short-term loans, long-term loans, and bills discounted in the TDB dataset. The mean of $\Delta LOANS$ is −5.2 percent, while the median is −3.5 percent (Table 2).

The main independent variables are the change in the expected rate of return on long-term bonds, $\Delta BOND RATE$, and bank-specific capital gains/losses as a result of changes in interest rates on bonds that banks have been holding, $BK\_CAPGAIN$. As a proxy for $\Delta BOND RATE$, we use the difference between 10-year forward interest rates, calculated in the following manner. We consider two forward rates: the forward rate observed in year $t−1$ for 10-year bonds starting in year $t$, and the forward rate observed in year $t−2$ for the same 10-year bonds starting in year $t$. We then take the difference between the two. If we denote the forward rate as $f_s(x,x + 10)$, where subscript $s$ is the year in which the forward contract is concluded and $x$ is the year in which the forward contract is executed, $\Delta BOND RATE$ is defined as:

$$\Delta BOND RATE = f_{t-1}(t, t + 10) - f_{t-2}(t, t + 10)$$

Thus, $\Delta BOND RATE$ captures the change between year $t−2$ and year $t−1$
in the expected return of the same 10-year bonds. Note that we use not the change in spot rates but the change in forward rates. Using forward rates enable us to correctly identify unanticipated changes in the expected returns on bonds, while spot rates may well be contaminated by contemporaneous macroeconomic conditions that affect banks’ lending behavior simultaneously. If they are indeed contaminated, the use of spot rates might result in a biased estimates of the portfolio balance channel. Table 2 shows that the mean of \( \Delta BONDRATE \) is \(-0.35\) percentage points, while the median is \(-0.45\) percentage points. Based on the model in Section 3, we expect that the coefficient on \( \Delta BONDRATE \) is negative if the substitution effect is larger than the income effect.

We calculate the bank-specific capital gains/losses stemming from banks’ exposure to interest rate risk via the holding of bonds with various maturities as follows:

\[
BK\_CAPGAIN = -\sum_s (\Delta BONDRATE\_SPOT_t(s) \times BK\_BOND_{t-1}(s) \times s) / BK\_TA_{t-1}
\]

where \( \Delta BONDRATE\_SPOT_t(s) \) is the change in the spot interest rate in year

\[12\] To be precise, we use 10-year implied forward rates, which are calculated from spot rates of various maturities observed in different years, based on the assumption that term structure is explained by expectation theory.
and $s$ represents the maturity of various spot rates. $BK_{BOND}_{t-1}(s)$ represents a bank’s holdings of bonds with maturity $s$ in year $t-1$, and $BK_{TA}_{t-1}$ is a bank’s total assets in year $t-1$, which are used to express changes in the value of bond holdings relative to the bank’s assets. Table 2 shows that the mean of $BK_{CAPGAIN}$ is 0.04 percent, while the median is 0.08 percent. Based on the model in Section 3, the coefficient on $BK_{CAPGAIN}$ should take a positive value if the net worth effect is present.

Figure 1 shows developments in the key variables of interest during the period 2002–2014. As can be seen in Figure 1(a), the median of $\Delta LOANS$ was negative and fairly stable during this period, although loans contracted at a faster rate during the period 2002–2004 when Japanese banks were reducing massive non-performing loans and again in 2009 and 2010 in the midst of the Great Depression.

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13 Banks disclose their bond holdings for each maturity based on the following categories: less than 1 year, 1–5 years, 5–10 years, and 10 years or more. Thus, to calculate $BK_{CAPGAIN}$, we use the median value of each category for the spot rate and maturity; that is, we use $s = 0.5, 3, 7.5,$ and 12 years respectively.

14 One may argue that bank-specific capital gains/losses also arise through banks’ holding of stocks if, for example, there is a negative correlation between bond prices and stock prices as predicted by the discounted cash flow model, so that lower long-term interest rates boost stock prices. Lower long-term interest rates may also increase bank net worth through the changes in the fair value of loans and deposits, which are usually recorded on a book-value basis, if the effective maturity (i.e., the time interval for changes in interest rates) of loans is longer than that of deposits. However, we do not have reliable data on the correlation of the prices of bonds and stocks held by each bank. Nor do we have data on the effective maturity of bank loans and deposits. Thus we abstract from these changes when calculating $BK_{CAPGAIN}$.
Recession. Unanticipated changes in the forward rate, $\Delta BOND RATE$, were mostly negative, except in 2005 and 2006 (Figure 1(b)). Finally, the median of banks’ capital gains on the bonds that it holds, $BK\_CAPGAIN$, was positive in 2002–2003 and 2008-2013 (Figure 1(c)).

We also use the following time-variant covariates that may affect $\Delta LOANS$. Regarding macroeconomic variables, the most important variable for our analysis is the expected rate of return of loans. We use the annual change in the average contract interest rate on new loans and bills discounted published by the Bank of Japan ($\Delta LOAN RATE$). Based on the prediction of our model in Section 3, we expect the coefficient on $\Delta LOAN RATE$ to be positive. In addition to $\Delta LOAN RATE$, we use the nominal GDP growth rate ($\Delta GDP$) and the annual percentage change of the Tokyo Stock Price Index ($\Delta TOPIX$).

As for variables representing bank characteristics, we use the bank capital-asset ratio, which is the bank net worth over total assets ($BK\_CAP$). We employ $BK\_CAP$ as a proxy for banks’ lending capacity. As mentioned above, the financial strength of Japanese banks was weak in the early 2000s due to the non-performing loan problem, which may have weakened loan supply. Further,
to take into account that the effect of bank net worth on bank loan supply may be non-linear we also include the square of this term ($BK_{CAP\_SQ}$). In addition, we use the bank liquidity ratio ($BK_{LIQ}$), the bank return on assets ($BK_{ROA}$), bank size as measured by the logarithm of total assets ($BK_{InTA}$), and a dummy variable that is equal to one if a bank is the main bank of a borrowing firm and zero otherwise ($BK_{MAIN}$).

Regarding firm characteristics, we use the firm capital-asset ratio ($F_{CAP}$), the liquidity ratio ($F_{LIQ}$), the return on assets ($F_{ROA}$), sales growth ($F_{\Delta SALES}$), firm size as measured by the logarithm of total assets ($F_{InTA}$), firm age (in logarithm, $F_{InAGE}$), and the logarithm of the number of banks that a firm transacts with ($F_{InNBANKS}$).

To deal with possible outliers in the TDB dataset, we winsorize the following firm-level variables at the upper and lower 0.5 percentiles: $\Delta LOANS$, $F_{CAP}$, $F_{LIQ}$, $F_{ROA}$, and $F_{\Delta SALES}$.

5. Results

In this section, we present our estimation results. Section 5.1 presents the main results on the portfolio balance channel and the bank balance sheet channel.
Section 5.2 examines the relative strength of the portfolio balance channel among banks. Section 5.3 presents the estimation results on the risk-taking channel.

5.1. Main results: Portfolio balance channel and bank balance sheet channel

Table 3 presents the main results of our empirical analysis. Columns (i), (ii) and (iii) respectively correspond to empirical specifications (8), (9), and (10) in Section 4.2, with the rows reporting the estimated coefficients and heteroskedasticity-robust standard errors clustered at the bank level.

Starting with the results in column (i), we find that the coefficient on $\Delta BOND RATE$, representing unexpected changes in the long-term forward rate, is negative and significant. The estimated coefficient implies that a 100-basis point decrease in the long-term forward rate increases firms’ loan growth rate by 1.6 percentage points. This result suggests that the substitution effect is larger than the income effect. Further, consistent with the theoretical model, the coefficient on $\Delta LOAN RATE$ is significantly positive. Turning to the coefficient on $BK\_CAPGAIN$, which measures the net worth effect, this is positive and significant. The estimated coefficient implies that a one standard
deviation increase in $BK\_CAPGAIN$, which corresponds to an increase in the ratio of bank capital to total assets by 0.18 percentage points, increases bank loan supply by 0.8 percentage points. Compared to the mean of $\Delta LOANS$, which is −5.2%, the net worth effect is of modest but not negligible economic significance.

Next, looking at the other covariates, the results are mostly consistent with our expectations. Of the remaining macroeconomic control variables, $\Delta TOPIX$ takes a significant positive coefficient, implying that the loan growth rate is higher when the stock market is doing well. As for bank characteristics, the coefficient on $B\_CAP$ is significantly positive while that on $B\_CAP\_SQ$ is significantly negative, indicating that the effect of bank net worth on loan supply is non-linear in that the positive marginal effect diminishes as the bank capital-asset ratio increases. Next, the coefficient on $BK\_MAIN$ is positive and significant, which suggests that a closer firm-bank relationship has a positive effect on the loan growth rate. Finally, all of the firm characteristics variables we employ have significant coefficients, indicating that the growth rate of loans from an individual bank is higher the higher a firm’s capital ratio, liquidity ratio,
ROA, and sales growth, the smaller and younger a firm, and the smaller the number of banks it transacts with.

Next, columns (ii) and (iii) respectively show the estimation results with year fixed effects and firm-year fixed effects. Note that the macroeconomic variables are dropped in column (ii), while the firm characteristics variables as well as the macroeconomic variables are dropped in column (iii). Thus, we cannot estimate the effect of $\Delta BOND RATE$ in these specifications. On the other hand, controlling for unobservable time-variant macroeconomic conditions and unobservable firm-level characteristics including firms’ loan demand allows us to more precisely estimate the effect of $BK\_CAPGAIN$ on loan supply. We find that the coefficient on $BK\_CAPGAIN$ is 3.57 in column (ii) and 4.65 in column (iii) compared to 4.60 in column (i). Thus, the coefficients are of a similar magnitude as that in column (i). However, because of larger standard errors $BK\_CAPGAIN$ is significant only at the 10 percent level in columns (ii) and (iii), which is lower than in column (i). The larger standard errors in columns (ii) and (iii) suggest that there may exist significant heterogeneity in firm and bank characteristics that affects the magnitude of the
effect of changes in bank net worth on loan supply. The role of heterogeneity in firm and bank characteristics is discussed in the following subsections.

5.2. Relative strength of the portfolio balance channel among banks

In this subsection, we investigate whether the portfolio balance channel is stronger for banks facing higher loan interest rates than those facing lower loan rates by interacting $\Delta BOND RATE$ with changes in bank-specific loan interest rates ($BK_{ALOAN RATE}$). $BK_{ALOAN RATE}$ represents the yearly change in the ratio of a bank’s loan interest income to total loans outstanding. We predict that the substitution effect from bonds to loans is stronger for banks facing higher expected returns on loans and therefore that the coefficient on $\Delta BOND RATE \times BK_{ALOAN RATE}$ is negative.

Column (iv) of Table 3 shows the estimation results. The coefficient on the interaction term $\Delta BOND RATE \times BK_{ALOAN RATE}$ is significantly negative, which is consistent with our prediction.

5.3. Results on the risk-taking channel

In this subsection, we examine the risk-taking channel. The model in Section 3 implies that a positive bank net worth shock affects banks’ risk taking capacity
and that banks’ supply of risky loans increases more than that of safe loans.\textsuperscript{15} Thus, in response to a bank-specific positive net worth shock stemming from banks’ exposure to interest rate risk, banks may be more aggressive in extending loans to riskier firms. To examine this possibility, we use firms’ size, capital-asset ratio, and TDB credit score as proxies for firms’ degree of riskiness. We assume that smaller firms, more leveraged firms with a lower capital-asset ratio, and firms with a lower TDB credit score are riskier. We construct a dummy variable that equals one if a firm’s total assets are smaller than the median, $dum\_F\_InTA\_SMALL$, and expect that the coefficient on $BK\_CAPGAIN \times dum\_F\_InTA\_SMALL$ is larger than that on $BK\_CAPGAIN \times (1 - dum\_F\_InTA\_SMALL)$ for larger firms. In a similar vein, we construct dummy variables $dum\_F\_CAP\_SMALL$ and $dum\_F\_SCORE\_LOW$ that equal one if the capital-asset ratio and TDB score of a firm is smaller than their sample median.

Table 4 displays the estimation results. We find significant positive

\textsuperscript{15} Ioannidou et al. (2015), Jimenez et al. (2014), and Paligorova and Santos (2017) find evidence for the risk-taking channel of monetary policy. Also see Aoki and Sudo (2012), who argue that a deterioration in banks’ net worth reduces their risk taking capacity and results in a rebalancing of banks’ portfolios towards government bonds. Meanwhile, Duchin and Sosyura (2014) report that U.S. banks that received government assistance from the Troubled Asset Relief Program shifted their asset allocation to riskier assets.
coefficients for firms that are smaller, have a lower capital-asset ratio, and have a lower TDB score (columns (i), (ii), and (iii)). These results indicate that the bank net worth effect is stronger for loans to riskier firms, and suggest the existence of the risk-taking channel.

6. Conclusion

Employing a unique and massive firm-bank loan-level panel dataset covering a variety of banks and firms in Japan during the period 2002–2014, this study investigated the effects of long-term interest rates on bank loan supply to firms. To disentangle the effects of interest rates on bank loan supply from those on bank loan demand, we incorporated firm-year fixed effects to control for time-varying unobservable loan demand. Our empirical analysis yielded the following results. First, a decrease in long-term interest rates led to an increase in banks’ loan supply, providing evidence for the existence of the portfolio balance channel, which consists of the net outcome of the substitution effect (the shift from government bonds to loans under the VaR constraint) and the income effect (slower loan growth due to the decrease in income from government bonds that tightens the VaR constraint). Second, we find that an
increase in banks’ net worth as a result of an increase in the value of bond holdings brought about by a decline in long-term interest rates led to an increase in loans to firms, providing evidence for the bank balance sheet channel. Third, we find that the effect of strengthened bank balance sheet is stronger in the case of loans to smaller, more leveraged, and less creditworthy firms, providing evidence for the risk-taking channel.

The analysis in this study raises a number of issues that remain to be addressed in future research. First, while we provide evidence for the existence of the portfolio balance sheet channel and the bank balance sheet channel (supply factors), how important they are in quantitative terms relative to demand factors (such as an increase in loan demand due to lower long-term interest rate) remains an open question. Our estimation results suggest that the economic impact of these channels is modest, but in order to gain a better quantitative understanding of the transmission of monetary policy it is necessary to decompose the sluggish loan growth during the lost decades in Japan into demand and supply factors in a more rigorous manner. Second, while we find that changes in long-term interest rates affect banks’ loan supply, such changes
in loan supply may not materially affect client firms’ real activities such as investment and employment if firms are not credit constrained due to the availability of other sources of funds. In order to assess the true significance of the two transmission channels, one has to know the elasticity with which borrower firms can switch between borrowing from banks and other sources of funds, which may be heterogeneous depending on firms’ and banks’ characteristics as well as the closeness of firm-bank relationships. Third, while we find evidence that a reduction in long-term interest rates led banks to particularly increase loan supply to credit-constrained and riskier firms, whether banks’ portfolio composition shifted toward riskier assets remains an open question. It may well be the case that the magnitude of the changes in banks’ portfolio composition differs across banks, so that one has to find a way to control for the aggregate loan demand that each bank faces in examining the shift in bank portfolios. How firms respond to loan supply shocks to their lender banks, how important bank loan supply shocks are for the economy, and how banks’ asset portfolios shift in response to changes in long-term interest rates are issues we leave for future research.
References


Khwaja, A. I., and A. Mian (2008). “Tracing the impact of bank liquidity shocks:


Table 1: Definition of variables

This table presents the definition of variables used in the main estimations (Table 3). All independent variables are as of 1 year prior (t-1) to the dependent variable $\Delta LOANS(t)$.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variable</strong></td>
<td></td>
</tr>
<tr>
<td>$\Delta LOANS$</td>
<td>Log change in firm $i$’s total loans outstanding from bank $j$.</td>
</tr>
<tr>
<td>$\Delta LOANS$</td>
<td>Loans outstanding include short-term loans, long-term loans, and bills discounted.</td>
</tr>
<tr>
<td><strong>Key independent variables</strong></td>
<td></td>
</tr>
<tr>
<td>$\Delta BONDRATE$</td>
<td>Difference between the forward rate observed in year $t-1$ for 10-year bonds starting in year $t$ and the forward rate observed in year $t-2$ for the same 10-year bonds starting in year $t$</td>
</tr>
<tr>
<td>$BK_{_CAPGAIN}$</td>
<td>Bank $j$’s capital gains/losses due to changes in prices of bonds held</td>
</tr>
<tr>
<td><strong>Macroeconomic controls</strong></td>
<td></td>
</tr>
<tr>
<td>$\Delta ALOANRATE$</td>
<td>Change in average interest rate of newly contracted loans including bills discounted</td>
</tr>
<tr>
<td>$\Delta GDP$</td>
<td>Change in Japan’s nominal gross domestic product</td>
</tr>
<tr>
<td>$\Delta TOPIX$</td>
<td>Log change in Tokyo Stock Price Index (TOPIX)</td>
</tr>
<tr>
<td><strong>Bank characteristics</strong></td>
<td></td>
</tr>
<tr>
<td>$BK_{_CAP}$</td>
<td>The ratio of bank $j$’s net worth over total assets</td>
</tr>
<tr>
<td>$BK_{_CAP_SQ}$</td>
<td>Squared value of bank $j$’s net worth ratio</td>
</tr>
<tr>
<td>$BK_{_LIQ}$</td>
<td>The ratio of bank $j$’s liquid assets over total assets. Liquid assets include cash and due from banks, call loans, government bonds, and local government bonds.</td>
</tr>
<tr>
<td>$BK_{_ROA}$</td>
<td>Bank $j$’s total net income over total assets</td>
</tr>
<tr>
<td>$BK_{_lnTA}$</td>
<td>The logarithm of bank $j$’s total assets</td>
</tr>
<tr>
<td>$BK_{_MAIN}$</td>
<td>1 if firm $i$ regards bank $j$ as its main bank, 0 otherwise</td>
</tr>
<tr>
<td><strong>Firm characteristics</strong></td>
<td></td>
</tr>
<tr>
<td>$F_{_CAP}$</td>
<td>The ratio of firm $i$’s net worth over total assets</td>
</tr>
<tr>
<td>$F_{_LIQ}$</td>
<td>The ratio of firm $i$’s liquid assets over total assets</td>
</tr>
<tr>
<td>$F_{_ROA}$</td>
<td>The ratio of firm $i$’s total net income over total assets</td>
</tr>
<tr>
<td>$F_{_ASALES}$</td>
<td>Log change in firm $i$’s gross sales</td>
</tr>
<tr>
<td>$F_{_lnTA}$</td>
<td>The logarithm of firm $i$’s total assets</td>
</tr>
<tr>
<td>$F_{_lnAGE}$</td>
<td>The logarithm of (1 plus firm $i$’s age)</td>
</tr>
<tr>
<td>$F_{_lnNBANKS}$</td>
<td>The logarithm of (1 plus the number of banks with which firm $i$ transacts)</td>
</tr>
</tbody>
</table>
Table 2: Summary statistics

This table presents summary statistics of the variables used in the main estimations (Table 3). Definitions of variables are provided in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Median</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variable</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∆LOANS</td>
<td>%</td>
<td>-5.21</td>
<td>66.18</td>
<td>-310.27</td>
<td>-3.50</td>
<td>321.89</td>
</tr>
<tr>
<td><strong>Key independent variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∆BONDRATE</td>
<td>% points</td>
<td>-0.35</td>
<td>0.32</td>
<td>-0.89</td>
<td>-0.45</td>
<td>0.21</td>
</tr>
<tr>
<td>BK_CAPGAIN</td>
<td>%</td>
<td>0.04</td>
<td>0.18</td>
<td>-1.53</td>
<td>0.08</td>
<td>1.59</td>
</tr>
<tr>
<td><strong>Macroeconomic controls</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∆LOANRATE</td>
<td>% points</td>
<td>-0.05</td>
<td>0.12</td>
<td>-0.23</td>
<td>-0.05</td>
<td>0.15</td>
</tr>
<tr>
<td>∆GDP</td>
<td>%</td>
<td>-0.60</td>
<td>1.87</td>
<td>-4.60</td>
<td>0.20</td>
<td>1.80</td>
</tr>
<tr>
<td>∆TOPIX</td>
<td>%</td>
<td>-4.41</td>
<td>16.85</td>
<td>-38.38</td>
<td>-2.34</td>
<td>38.53</td>
</tr>
<tr>
<td><strong>Bank characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BK_CAP</td>
<td>%</td>
<td>4.55</td>
<td>1.43</td>
<td>0.38</td>
<td>4.50</td>
<td>15.54</td>
</tr>
<tr>
<td>BK_CAP_SQ</td>
<td>%</td>
<td>22.72</td>
<td>14.34</td>
<td>0.15</td>
<td>20.25</td>
<td>241.45</td>
</tr>
<tr>
<td>BK_LIQ</td>
<td>%</td>
<td>22.49</td>
<td>7.20</td>
<td>4.62</td>
<td>21.08</td>
<td>72.95</td>
</tr>
<tr>
<td>BK_ROA</td>
<td>%</td>
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<td>-1.83</td>
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</tr>
<tr>
<td>BK_InTA</td>
<td>Mil. yen</td>
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<td>1.85</td>
<td>10.69</td>
<td>15.58</td>
<td>19.02</td>
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<tr>
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<td>0.48</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Firm characteristics</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F_CAP</td>
<td>%</td>
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<td>19.32</td>
<td>-99.41</td>
<td>20.00</td>
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</table>
Figure 1: Developments in key variables

These figures present developments in key variables used in the main estimations (Table 3). Definitions of variables are provided in Table 1 and in the text.

(a) Bank loan growth rate ($\Delta LOANS$)

(b) 10-year forward rates ($\Delta BOND RATE$)

(c) Banks’ capital gains ($BK\_CAPGAIN$)
### Table 3: Estimation results for the portfolio balance channel and the bank balance sheet channel

This table presents the estimation results on bank loan growth, $\Delta LOANS$, controlling for various covariates and fixed effects outlined in the text. ***, **, * indicate significance at the 1, 5, and 10% level, respectively. Standard errors in parentheses are heteroskedasticity-robust standard errors clustered at the bank level.

<table>
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<tr>
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<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
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<td><strong>Key independent variables</strong></td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.84)</td>
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<td></td>
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<tr>
<td>$BK_{CAPGAIN}$</td>
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<td>4.65</td>
<td>4.62</td>
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<td></td>
<td>(1.36)</td>
<td>(2.05)</td>
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<td>(2.74)</td>
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<td>$\Delta BOND RATE \times BK_{ALOAN RATE}$</td>
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<td>$\Delta LOAN RATE$</td>
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<td>$\Delta GDP$</td>
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<td>$\Delta TOPIX$</td>
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<td>(0.03)</td>
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<td><strong>Bank characteristics</strong></td>
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<td>(0.68)</td>
<td>(0.70)</td>
<td>(1.00)</td>
<td>(1.17)</td>
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<td>$BK_{_CAP _SQ}$</td>
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<td>-0.31</td>
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<td>(0.08)</td>
<td>(0.08)</td>
<td>(0.11)</td>
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<td>(0.04)</td>
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<td>Year</td>
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<td>379,989</td>
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<td>379,989</td>
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<td>0.04</td>
<td>0.21</td>
<td>0.21</td>
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Table 4: Estimation results for the risk-taking channel: Interaction terms with banks’ capital gains

This table presents the estimation results on bank loan growth, $\Delta LOANS$, when interaction terms between banks’ capital gains, $BK\_CAPGAIN$, and firm characteristics are included. Columns (i)-(iii) show the results when $BK\_CAPGAIN$ is interacted with a firm’s asset size, capital-asset ratio, and TDB score, respectively. Other independent variables included in the estimations are bank characteristics variables, firm-year fixed effects, and bank fixed effects (as in the specification in column (iii) of Table 3). ***, **, * indicate significance at the 1, 5, and 10% level, respectively. Standard errors in parentheses are heteroskedasticity-robust standard errors clustered at the bank level.

<table>
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<th>Interaction term with $BK_CAPGAIN$</th>
<th>$dum_F_lnTA_small$</th>
<th>$dum_F_CAP_small$</th>
<th>$dum_F_SCORE_low$</th>
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<tr>
<td>Small (low)</td>
<td>15.52 ***</td>
<td>9.13 ***</td>
<td>6.79 *</td>
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<td>(5.04)</td>
<td>(2.84)</td>
<td>(3.60)</td>
</tr>
<tr>
<td>Large (high)</td>
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<td>-2.38</td>
<td>3.98</td>
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<tr>
<td></td>
<td>(2.91)</td>
<td>(4.64)</td>
<td>(3.46)</td>
</tr>
</tbody>
</table>

Bank characteristics: YES YES YES

Fixed effects:
- Firm-year: YES YES YES
- Bank: YES YES YES

Observations: 379,989 379,989 379,109

Adjusted $R^2$: 0.21 0.21 0.21
Appendices for Online Publication
Appendix A. Background figures for the developments in monetary policy and bank portfolios in Japan

Figure A–1: Monetary policy measures and long-term interest rates in Japan

This figure presents development in monetary policy measures and long-term interest rates. Monetary policy measures are the uncollateralized overnight call rate and the amount of Japanese government bonds (JGBs) held by the Bank of Japan. Long-term interest rates are represented by the 10-year yield on newly issued JGBs.

Sources: Bank of Japan, Japan Bond Trading Co., Ltd.
Figure A–2: Japanese banks’ asset portfolios and asset returns
These figures present developments in Japanese banks’ asset portfolios (bonds and loans) and asset returns using aggregate data. Panel (a) shows the ratio of Japanese government bond (JGB) holdings to total loans outstanding, (b) shows the annual rate of change in loans outstanding to corporations, and (c) shows the average interest rate on newly contracted loans including bills discounted. The return on JGBs is presented in Figure 1.
Source: Bank of Japan

(a) Banks’ asset portfolios: Bonds/loans ratio

(b) Annual growth rate of loans to corporations

(c) Average loan interest rate
Appendix B. Bank portfolio selection model

To derive theoretical predictions on the effect of long-term interest rates on bank lending, we construct, as mentioned in the text, a simple bank portfolio selection model. In this Appendix, we provide a detailed analysis of banks’ portfolio selection with the VaR constraint, from which we abstracted in the text.

As in Section 3, the VaR constraint is given by the following inequality:

\[-(l_L L + l_B B) \leq N \quad \text{(A.1)}\]

where \[l_L = \frac{\mu_L - r_D - \pi L}{\sigma_L} \quad \text{and} \quad l_B = \frac{\mu_B - r_D - \pi B}{\sigma_B},\] which represent the unexpected losses on loans and bonds at the time of stress. We assume that \[l_L \quad \text{and} \quad l_B \] are negative.

From the Kuhn-Tucker conditions, we obtain

\[L^{**} = L^* + \lambda L' \quad \text{(A.2)}\]

\[B^{**} = B^* + \lambda B' \quad \text{(A.3)}\]

where \[L^* \quad \text{and} \quad B^* \] are the optimal amount of loans and bonds without the VaR constraint, which are given by equations (A.4) and (A.5) below.

\[L^* = \frac{1}{\gamma} \left( \frac{\mu_L - r_D}{\sigma_L^2} \right) \quad \text{(A.4)}\]

\[B^* = \frac{1}{\gamma} \left( \frac{\mu_B - r_D}{\sigma_B^2} \right) \quad \text{(A.5)}\]
\( \lambda \) represents the shadow price of a bank’s capital \( N \) (i.e., the Lagrange multiplier associated with equation (A.1)), and \( L’ \) and \( B’ \) are given by

\[
L’ = \frac{l_L}{\gamma \sigma_L^2} < 0 \quad (A.6)
\]

\[
B’ = \frac{l_B}{\gamma \sigma_B^2} < 0 \quad (A.7)
\]

Substituting (A.2) and (A.3) into (A.1) yields

\[
\lambda = -\frac{l_l L’ + l_B B’ + N}{l_l L’ + l_B B’} > 0 \quad (A.8)
\]

Thus, from equations (A.2) and (A.3), it can be easily seen that the optimal amount of loans and bonds under the VaR constraint, \( L^{**} \) and \( B^{**} \), is smaller than the optimal amount of loans and bonds without the VaR constraint, \( L^* \) and \( B^* \).

Let us now consider the comparative statics of the effects of a change in \( \mu_B \) on \( L^{**} \). From equation (A.2), we have

\[
\frac{\partial L^{**}}{\partial \mu_B} = L’ \frac{\partial \lambda}{\partial \mu_B} = L’ \left[ \theta_1 \frac{\partial l_B}{\partial \mu_B} + \theta_2 \frac{\partial B’}{\partial \mu_B} \right] \quad (A.9)
\]

\[
\theta_1 = \frac{-B^{**}}{l_l L’ + l_B B’} < 0 \quad (A.10)
\]

\[
\theta_2 = \frac{-(1 + \lambda/r_d)l_B}{l_l L’ + l_B B’} > 0 \quad (A.11)
\]
Because $\frac{\partial l_B}{\partial \mu_B} > 0$ and $\frac{\partial B^*}{\partial \mu_B} > 0$, the first term $\theta_1 \frac{\partial l_B}{\partial \mu_B}$ in equation (A.9) is negative, while the second term $\theta_2 \frac{\partial B^*}{\partial \mu_B}$ is positive. The first term shows that an increase in $\mu_B$ reduces the unexpected loss on bonds and reduces the Lagrange multiplier $\lambda$, therefore relaxes the VaR constraint. It therefore has a positive impact on $L^{**}$. The second term shows that an increase in $\mu_B$ raises the amount of bond holdings, which in turn tightens the VaR constraint (increases the Lagrange multiplier). Thus, it has a negative impact on $L^{**}$. The overall effect of $\mu_B$ on $L^{**}$ depends on the relative magnitude of these two opposing effects.

Next, we consider the comparative statics of the effects of a change in $N$ on $L^{**}$.

$$\frac{\partial L^{**}}{\partial N} = L' \frac{\partial \lambda}{\partial N} = - \frac{L'}{l_b L' + l_b B'} > 0 \quad \text{(A.12)}$$

Because an increase in $N$ always relaxes the VaR constraint, it has a positive impact on $L^{**}$.

Next, we extend our analysis above (the benchmark case) to a three-asset model. Assume that there are two kinds of loans: safe loans ($L$) and risky loans ($R$). We define the riskiness of loans in terms of the mean, standard deviation, and
Sharpe ratio of the return on loans. Specifically, we assume that the return on risky loans has a higher mean and higher standard deviation than safe loans, while the Sharpe ratio of risky loans is lower than that of safe loans:

\[ \mu_L < \mu_R \]  
\[ \sigma_L < \sigma_R \]  
\[ \frac{\mu_L - r_D}{\sigma_L} \geq \frac{\mu_R - r_D}{\sigma_R} \]  

Equation (A.15) implies that loans are risky if they do not offer a sufficiently large risk premium to compensate for their return volatility.

As in the benchmark case, we assume that the returns of all three assets are independent from each other. Banks’ profits and balance sheets therefore take the following form:

\[ \pi = r_L L + r_R R + r_B B - r_D D \]  
\[ \text{s.t.} \quad L + R + B = D + N \]  

The VaR constraint takes the following form:

\[ (\mu_L - n\sigma_L - r_D)L + (\mu_R - n\sigma_R - r_D)R + (\mu_B - n\sigma_B - r_D)B + r_D N \geq 0 \]  

Similar to the benchmark case, banks’ choose \( L, R \) and \( B \) to maximize (4) subject to the VaR constraint (A.18). The optimal portfolios are given by
where $R^*$ and $R'$ are respectively defined as:

$$R^* = \frac{1}{\gamma} \left( \frac{\mu_R - r_D}{\sigma_R^2} \right)$$ \hspace{1cm} (A.22)

$$R' = \frac{l_R}{\gamma \sigma_R^2} < 0$$ \hspace{1cm} (A.23)

$$l_R = \frac{\mu_R - r_D - n\sigma_R}{r_D}$$ \hspace{1cm} (A.24)

The Lagrange multiplier $\lambda$ is [now] given by

$$\lambda = -\frac{l_L L' + l_R R^* + l_R B^* + N}{l_L L' + l_R R' + l_R B'} > 0$$ \hspace{1cm} (A.25)

We are interested in how the ratio of riskier loans to safer loans changes as banks’ net worth and long-term interest rates change. Note that equations (A.19) and (A.20) imply that

$$\frac{R^{**}}{L^{**}} = \frac{\sigma_L^2}{\sigma_R^2} \left( \frac{\mu_R - r_D + \lambda l_R}{\mu_L - r_D + \lambda l_L} \right)$$ \hspace{1cm} (A.26)

Inspection of equation (A.26) reveals that, under assumption (A.15),

$$\frac{\partial [R^{**}/L^{**}]}{\partial \lambda} < 0$$ \hspace{1cm} (A.27)

Recall that the Lagrange multiplier represents the shadow value of banks’ net worth under the VaR constraint. Equation (A.27) implies that when the VaR
constraint loosens banks invest in riskier loans with a lower Sharpe ratio.

Similar to equation (A.8) in the benchmark case, equation (A.25) implies that

$$\frac{\partial \lambda}{\partial N} < 0$$  \hspace{1cm} (A.28)

In other words, when banks’ net worth increases, the VaR constraint loosens.

Combining equations (A.27) and (A.28), we obtain

$$\frac{\partial [R^{**}/L^{**}]}{\partial N} = \frac{\partial [R^{**}/L^{**}]}{\partial \lambda} \frac{\partial \lambda}{\partial N} > 0$$  \hspace{1cm} (A.29)

This implies that when banks’ net worth increases due to a capital gain from long-term bonds, they increase loans to riskier firms more than loans to safer firms.

Next, we analyze how $R^{**}/L^{**}$ changes when the long-term interest rate declines. Note that

$$\frac{\partial [R^{**}/L^{**}]}{\partial \mu_B} = \frac{\partial [R^{**}/L^{**}]}{\partial \lambda} \frac{\partial \lambda}{\partial \mu_B}$$  \hspace{1cm} (A.30)

where $\frac{\partial [R^{**}/L^{**}]}{\partial \lambda} < 0$ (equation (A.27)). Similar to the benchmark case in equation (A.9), the sign of $\frac{\partial \lambda}{\partial \mu_B}$ depends on the size of the two offsetting effects (the relative values of the first term and the second term):

$$\frac{\partial \lambda}{\partial \mu_B} = \theta_1 \frac{\partial l_B}{\partial \mu_B} + \theta_2 \frac{\partial B^*}{\partial \mu_B}$$  \hspace{1cm} (A.31)

where $\theta_1 < 0$ and $\theta_2 > 0$ are defined analogously to equations (A.10) and
(A.11). Therefore, the effect of the decline in the long-term interest rate on \( R^{\ast\ast}/L^{\ast\ast} \) is ambiguous.